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A DYNAMIC MODEL FOR THE ALLOCATION OF AIRSTRIKES AGAINST A LINES-OF-COMMUNICATION NETWORK

by

Larry Reginald Capps

United States
Naval Postgraduate School

THESIS

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Larry Reginald Capps

SEP 1970

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A Dynamic Model for the Allocation of Airstrikes Against a Lines-of-Communication Network

by

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

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ABSTRACT

A computer model is presented for determining the daily allocation of airstrikes for interdicting a lines-of-communication (LOC) network assuming an exponential damage function. The allocation of sorties to arcs in the network is based on the assumptions that (1) flow of supplies is restricted by network capacity and (2) parameter values for each arc are independent. Upper and lower flow bounds, cost per unit flow, maximum repair time and cost, and the vulnerability parameter for each arc are required as input data. The model selects that arc to strike which maximizes the repair cost plus the product of the increase in minimum cost circulation flow and repair time. The procedure is programmed in daily cycles, allowing repair on interdicted arcs. A sample problem and all documentation necessary for duplicating the computer program are given.

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TABLE OF SYMBOLS AND ABBREVIATIONS

(i,j)	arc between nodes i and j
C_{ij}	cost of one unit of flow on arc (i.j)
CRT_{ij}	cumulative repair time for arc (i,j)
d_{ij}	damage on arc (i.j) caused by n_{ij} strikes
D_{ij}	total damage on arc (i,j) over one or more days
F'	cost of a minimum cost circulation flow after arc (i,j) has been struck
k_{ij}	cost to repair the arc damage caused by n_{ij} strikes
K_{ij}	maximum cost to restore an arc from its lower flow bound to its upper flow bound
L_{ij}	lower flow bound on arc (i,j)
M_{ij}	actual flow on arc (i,j)
n_{ij}	the number of strikes allocated to arc (i,j) in one day
N	maximum number of strikes that can be allocated in any one day
r_{ij}	amount of repair completed on arc (i,j) in one day
s	the Lagrange Multiplier
t_{ij}	time to repair the arc damage caused by n_{ij} strikes
T_{ij}	maximum time to restore an arc from its lower flow bound to its upper flow bound
U_{ij}	initial upper flow bound (maximum capacity) on arc (i,j)
$\bar{U}_{ij}$	upper flow bound on arc (i,j) at any time
V_{ij}	vulnerable flow capacity of arc (i,j), i.e., $V_{ij} = U_{ij} - L_{ij}$
VP_{ij}	vulnerability parameter for arc (i,j)

I. INTRODUCTION

A. GENERAL

The ability of a military force to conduct sustained combat operations any distance from its base of supply may be limited by the land lines-of-communication (LOC) network over which its supplies must flow. This is particularly true if air and sea resupply are not feasible, or the opposing force has superiority over these alternatives. The amount and type of material required to support a military force are determined by the nature of combat operations and the intensity of combat. A large conventional force would require more pounds-per-man-day than a small guerilla force capable of obtaining a portion of its requirements in the area in which it operates. Undoubtedly, however, a requirement will exist for supplies in excess of what can be obtained locally or what can be transported organically by the combat force.

The land LOC network then becomes an important military target. Flow of military material over the network can be reduced by any of three alternatives: (1) attacking the transport capability, i.e., vehicles or rolling stock, (2) attacking manufacturing and supply centers, thereby reducing the amount of materiel available, or (3) attacking the road/rail LOC network over which the materiel must flow. Certainly, a combination of these alternatives would be most desirable. However, political considerations might preclude attacking manufacturing and supply centers; and terrain, weather, and lack of intelligence information might limit the effectiveness of attempting

to attack convoys moving over the network.

Such a scenario is particularly applicable to the present war in Southeast Asia and is a general characteristic of limited warfare. The North Vietnamese neither possess a sophisticated air resupply capability, nor can they openly move supplies by sea to support their forces operating in South Vietnam. They do, however, enjoy immunity from attack on all their facilities north of the 17th Parallel. Therefore, the supply of materiel to their forces operating in the South is primarily limited by the transportation system over which this materiel must flow.

The objective of an interdiction campaign by airstrike must then be to systematically reduce the capacity of road or rail segments which will ultimately restrict the amount of materiel that can flow through the network.

B. AVAILABLE LITERATURE

Several authors have published works on the problem of interdicting a lines-of-communication network. Durbin [3] has constructed a model for determining maximum cargo flow as a function of available transport vehicles. His method sequentially selects and interdicts the most vital segment in the network until flow is reduced to zero or a specific number of segments have been destroyed. The model operates in daily cycles and allows repair on interdicted segments.

Wollmer [11, 14, 15] has developed a procedure to determine the n most vital segments in a network and to evaluate the effect

of removing the most vital segment. In the latter work, road capacity as a function of the number of strikes is reduced (1) by a known quantity and then (2) by a quantity that is a random variable.

Wollmer [12, 13] has also developed algorithms and a model for evaluating a targeting strategy whose aim is the greatest reduction in network flow. The model is programmed in daily cycles with the user specifying the number of days and strikes.

Mustin [9] uses the topological dual to represent a lines-of-communication network and assumes the decrease in capacity of a road or rail segment has a fixed deterministic rate which is linear between its upper and lower bounds.

Nugent [10] also uses the topological dual for network representation, but assumes the reduction in capacity of a road or rail segment is exponential with decreasing marginal returns. His model is constrained by the number of available sorties and does not allow for the dynamics of the daily allocation problem.

C. ALLOCATION OF EFFORT

Many techniques have been devised to aid in solving the problem of allocating limited resources. Hancock [8] provides a reference on classical optimization techniques; Danskin [1, 2] considers the theory of maxima and minima and its application to weapons systems allocation; and Koopman [6, 7] considers a similar problem in the allocation of search and screening effort. The problem facing the strike planner is to determine what road

or rail segments in the network to attack and how many strikes to allocate to each selected segment. Numerous considerations must be weighed in order to make this determination: (1) the number of sorties available on a given day, (2) the probability that a strike against a particular segment is successful, (3) the expected cost to the enemy in time and effort to repair an interdicted segment, (4) the expected cost of aircraft lost in striking a defended segment, and (5) the expected reduction in capacity if a strike is successful.

The obvious procedure in making the allocation of N sorties is to attack those segments (at a level of n_{ij} each) that maximize the ratio of benefits to cost. As will be shown later, Gibbs' Lemma [2] provides the mathematical basis for determining this allocation.

II. OBJECTIVE AND SCOPE

The objective of this paper is to present a model for determining the daily allocation of airstrikes for interdicting a lines-of-communication network assuming an exponential damage function. The model seeks to allocate a maximum of N (specified) sorties per day to those arcs (segments) which yield the greatest reduction in flow capacity and the greatest time and cost to repair. Two options are provided that will allow the strike planner to (1) specify the number of strikes per day and the number of days in the interdiction campaign, or (2) select a percentage of the maximum possible flow to be reduced. Using the second option, the maximum available number of sorties per day and the number of campaign days are specified.

Certain information is assumed to be available to the strike planner for each arc in the LOC network:

(1) Upper flow bound - maximum capacity in units per day.

(2) Lower flow bound - minimum capacity in units per day below which the flow cannot be reduced.

(3) Maximum repair time - the time required to repair the maximum possible damage on an arc.

(4) Maximum repair cost - the cost to repair the maximum possible damage on an arc.

(5) Vulnerability parameter - the probability that a sortie is successful. The strike planner can manipulate this parameter by providing his own definition of "successful."

The vulnerability of any arc, V_{ij}, is defined as the upper flow bound minus the lower flow bound, i.e., $U_{ij}-L_{ij}$. A necessary assumption is that the vulnerability, repair cost, and repair time are independent of the same parameters for any other arc.

By assuming an exponential damage function, the reduction in capacity d_{ij}, for any arc (i,j) caused by n_{ij} strikes is:

$$d_{ij} = V_{ij}[1 - \exp(-VP_{ij}\ n_{ij})] \tag{1}$$

Arc repair time and cost are linear functions of the amount of damage caused by n_{ij} strikes:

$$t_{ij} = (d_{ij}/V_{ij})T_{ij} \tag{2}$$

$$k_{ij} = (d_{ij}/V_{ij})K_{ij} \tag{3}$$

The amount of arc repair that can be completed in any one day is a function of the computed cumulative arc repair time and the total damage on the arc:

$$r_{ij} = (1/CRT_{ij})\ D_{ij} \tag{4}$$

The value of allocating n_{ij} strikes per day to arc (i,j) is determined by:

$$\text{Value}_{ij} = (F' - F)\ t_{ij} + k_{ij} \tag{5}$$

The model daily selects and interdicts those arcs that maximize (5), performs repair as indicated by (4), and steps to the next day. The procedure has been programmed in FORTRAN IV for use on the IBM 360/67 computer.

III. NETWORK REPRESENTATION OF LINES-OF-COMMUNICATION

A. NETWORK STRUCTURE

A transportation system, whether single or multimode, can be represented by a connected graph of nodes and directed arcs. For example, a highway system would consist of roads (arcs) and the intersection of roads (nodes). If a road is one-way, the beginning node of an arc is the point where traffic enters the segment and the ending node is the point where traffic leaves the segment. If a road is two-way, it is represented by two arcs, one in each direction.

The LOC user attempts to send units of flow, expressible in vehicles per day, tons per day, etc., from the source to the sink. The source of flow could be a rear-area supply center and the sink could be the battlefront or a forward-area supply center. If the user is constrained by the capacity of the transportation system, it is assumed that he will attempt to maximize the total flow of cargo through the network at a minimum cost.

Consider the example LOC network shown in Fig. 1. Suppose the user wishes to send flow from node AA to node ZZ. Node AA is the source and node ZZ is the sink. The route (AA,A,K,L,ZZ) is one of many possible routes over which the flow can be sent. Associated with each arc are certain parameters: lower bound on flow, upper bound on flow (flow capacity), and the cost per unit flow. For arc (K,L) the following parameters are given:

Lower bound (L_{ij}) 0
Upper bound (U_{ij}) 400
Flow cost (C_{ij}) 7

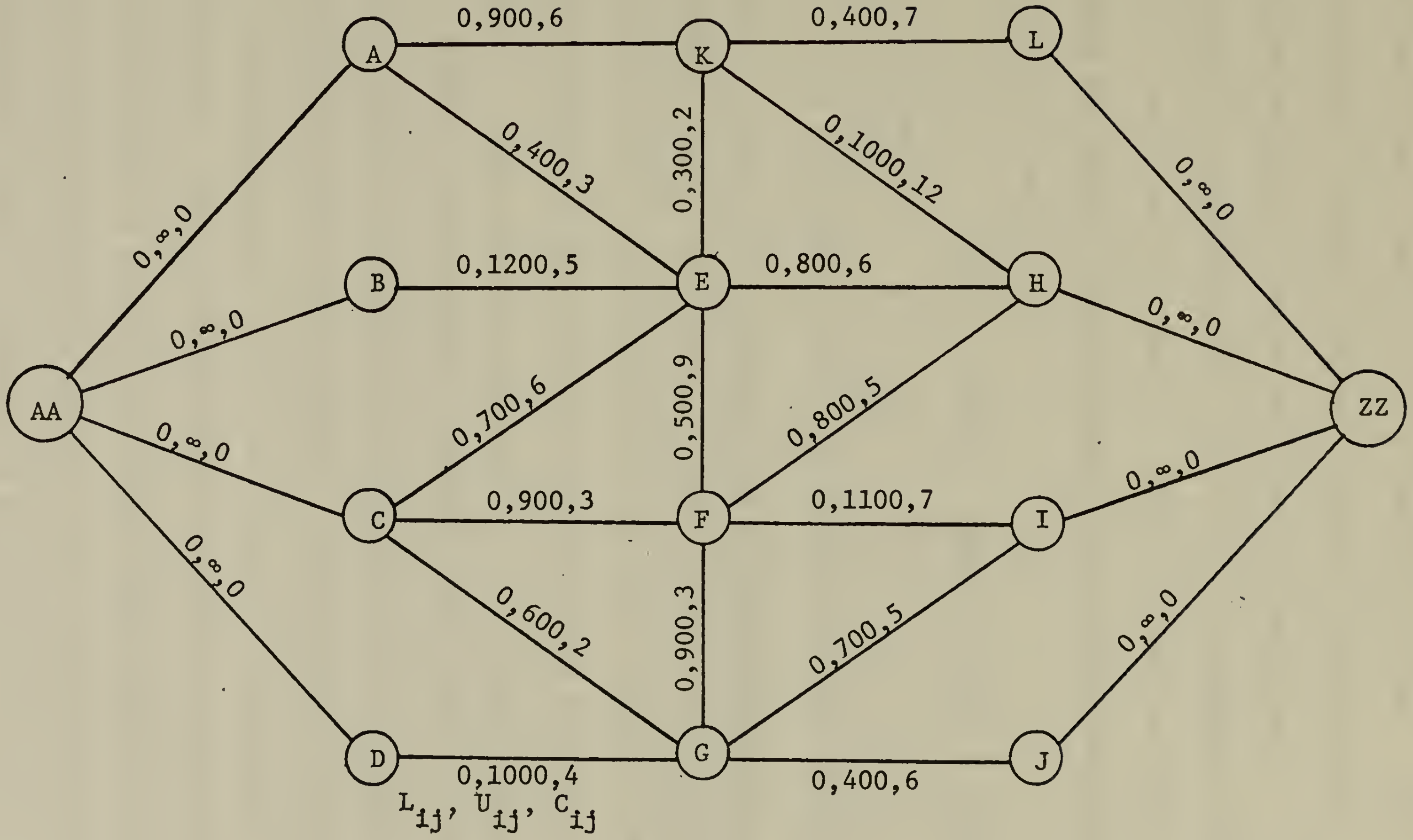


Figure 1. An Example Network

These parameters imply that between 0 and 400 units of flow per time unit can be sent over (K,L) at a cost of 7 per unit. This cost can be expressed in terms of dollars per ton of cargo, dollars per vehicle, manpower units, or any other definable cost. If the arc is used to its capacity, i.e., $M_{ij} = U_{ij}$, it would cost 2800 per time unit to send flow over this arc. The total cost per time unit of sending 400 units of flow from node AA to node ZZ over route (AA,A,K,L,ZZ) would be:

$$\sum_{\substack{\text{all } (i,j) \\ \text{in} \\ \text{the route}}} M_{ij}C_{ij} = 400(0) + 400(6) + 400(7) + 400(0) = 4200$$

It is important to note that the flow out of a node equals the flow into a node. This conservation of flow implies there can be no storage at nodes. Therefore, for most general networks, if x units of flow are sent from the source to the sink, then any flow pattern that accomplishes this will have net flows of x units out of the source, x units into the sink, and zero on all other nodes.

In actual LOC networks, there may be more than one source and/or sink. This can be handled, provided any source can supply any sink, by adding artificial nodes and arcs. This allows the user to convert the multi-source, multi-sink problem to a single-source, single-sink problem. The artificial arcs are used to connect the true sources (sinks) to an artificial super source (sink). Note in Figure 1 that arcs (AA,A), (AA,B), (AA,C), and (AA,D) can be considered artificial arcs connecting the true sources A, B, C, and D to the single artificial super source AA. Similarly, arcs (L,ZZ),

(H,ZZ), (I,ZZ), and (J,ZZ) can be considered artificial arcs connecting the true sinks L, H, I, and J to the artificial super sink ZZ. The flow on an artificial arc, e.g., (AA,A), can be considered the rate that cargo leaves the true source A. The capacity for artificial arcs is usually infinite, but could represent the maximum supply rate at a facility such as a supply center or warehouse. The lower flow bound and cost on artificial arcs are normally zero. Artificial arcs are not considered vulnerable to attack.

Actual flow on an arc is designated M_{ij} if the flow is from node i to node j and M_{ji} if the flow is in the reverse direction. Normally, flow is assumed to be from the source to the sink but can be in either direction on intermediate two-way arcs.

U_{ij}, $\bar{U}_{ij}$, and L_{ij} represent the upper flow bound, the flow capacity at any time, and the lower flow bound, respectively, for any arc (i,j). The following relationship must hold for these parameters:

$$0 \leq L_{ij} \leq M_{ij} \leq \bar{U}_{ij} \leq U_{ij}$$

The maximum possible flow through the network can be determined by using the maximal-flow minimal-cut theorem [4]. It states that the maximum flow in a network is equal to the minimum value of all cut-sets, where a cut-set is defined as a set of arcs that separate the source from the sink. The value of a cut-set is defined as the sum of the capacities of all arcs in the cut-set. In the example network, Figure 2, arcs (A,K), (K,E), (E,H), (E,F), (C,F), (C,G), and (D,G) form the minimum cut-set with a value of 5000 units.

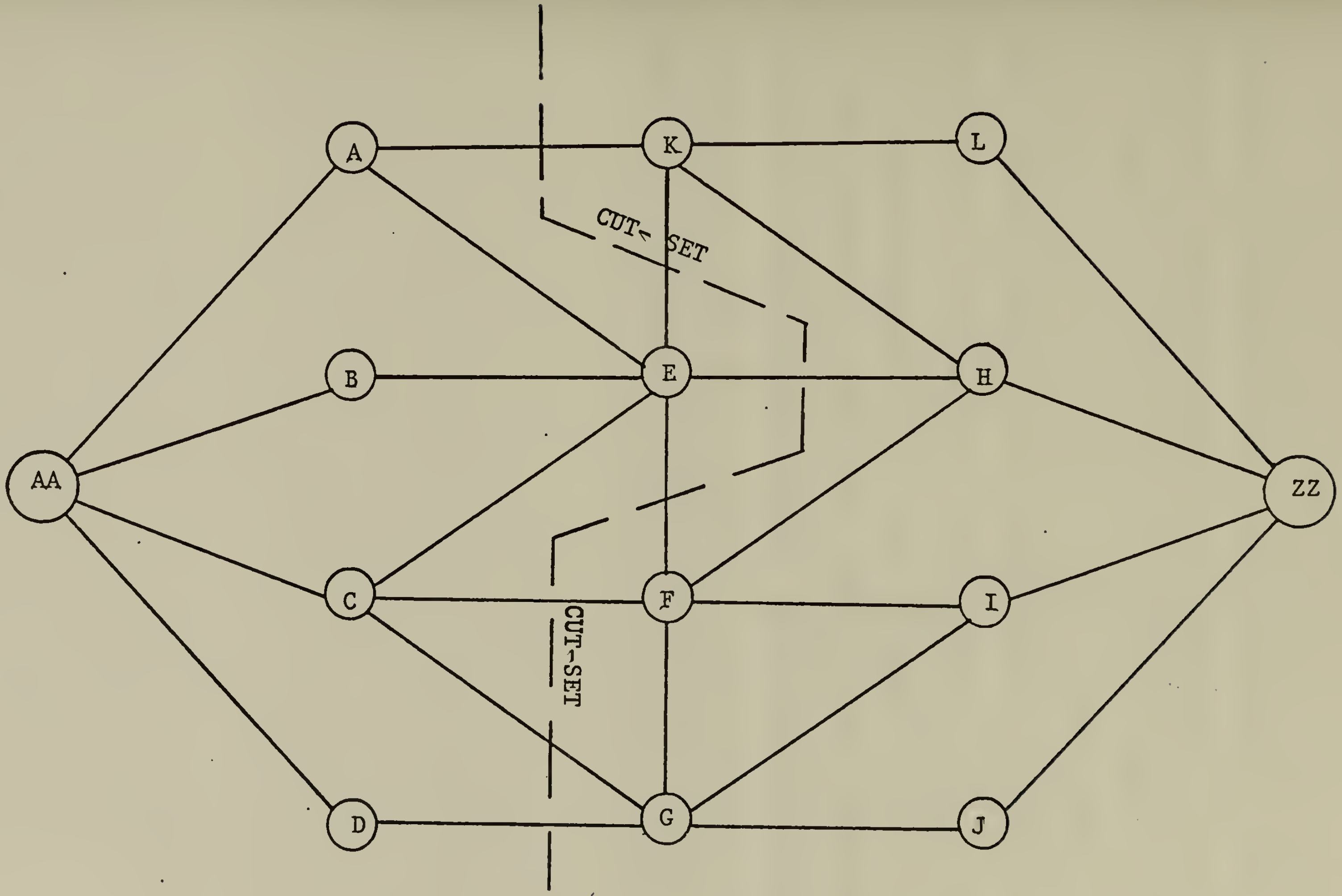


Figure 2. Minimum Cut-Set for Example Network

B. CIRCULATION FLOWS IN NETWORKS

As previously stated, if the LOC user is constrained by the capacity of the network, it is assumed he wishes to maximize the total flow through the network at a minimum cost. Such a problem is efficiently handled by the "Out-of-Kilter" algorithm developed by Fulkerson [4]. The network is restructured by adding a "universal" arc from the super sink to the super source as in Fig. 3. This artificial arc is assigned an upper flow bound greater than or equal to the maximum possible flow through the network (usually infinite), a lower bound of zero, and a unit flow cost less than the flow cost on any arc (usually minus infinity). Hence, the problem of maximizing source-sink flow is converted to one of finding a minimum cost circulation flow by maximizing flow on the universal arc.

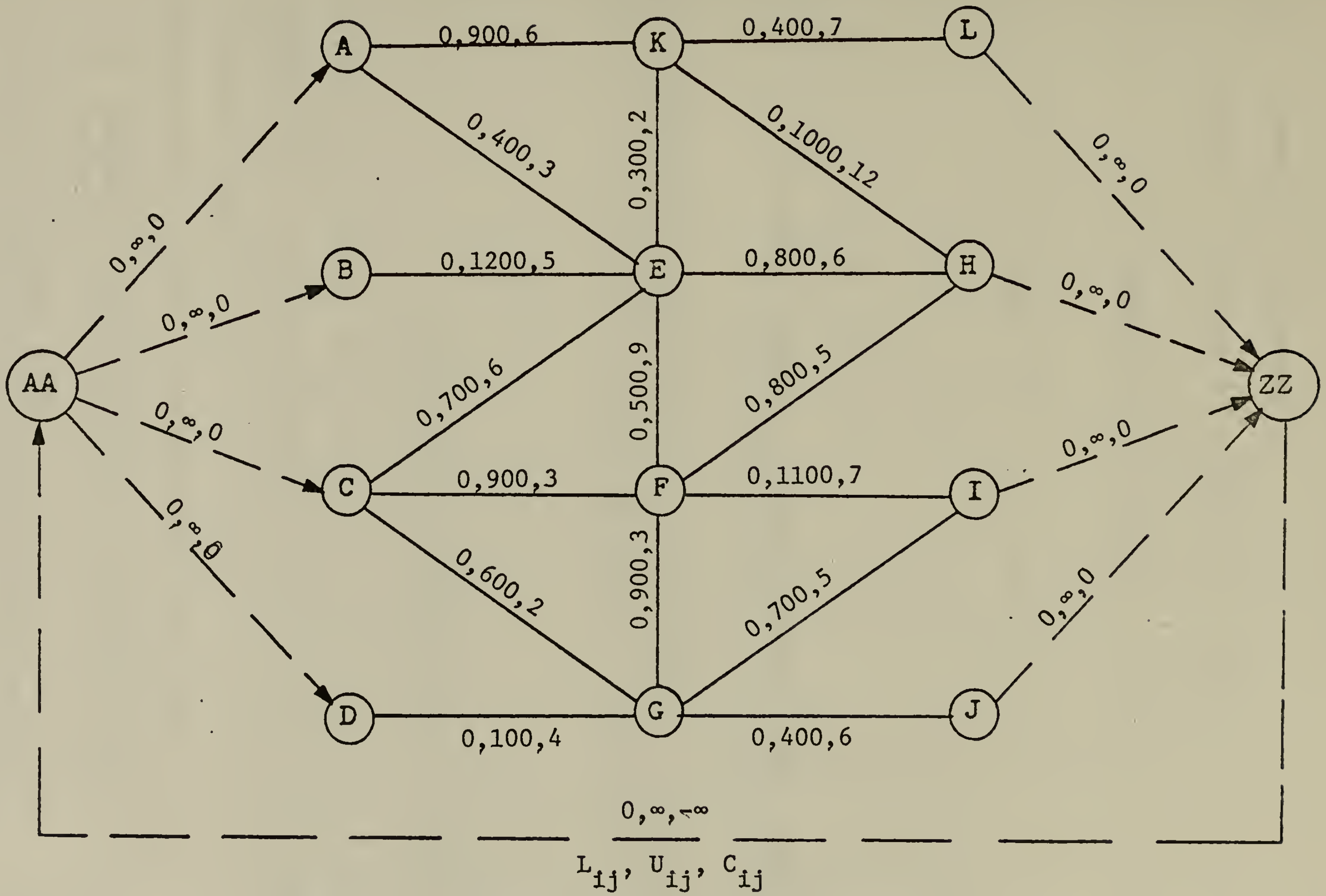


Figure 3. Example Network with Universal Arc

IV. MODEL OPERATION

A. MATHEMATICAL PRELIMINARIES

Assume: (1) The set $X^* = (x_1^*, x_2^*, \ldots, x_n^*)$

maximizes $\sum_{i=1}^{n} f_i(x_i)$

subj. to $\sum_{i=1}^{n} x_i = X \; ; x_i \geq 0$

(2) The $f_i(x_i)$ are continuous and differentiable

(3) $f_i''(x_i) \leq 0$

Note: These are necessary and sufficient conditions for a global maximum since the objective function is concave and the constraints are convex.

Then there exists a scalar $s > 0$ such that:

(1) $f_i'(x_i) = s$ for $x_i^* > 0$

(2) $f_i'(x_i) < s$ for $x_i^* = 0$

Note: The Lagrange Multiplier, s, can be interpreted as the implicit marginal price of allocating an additional unit of resource.

The above is known as Gibbs' Lemma and is discussed in more detail by Danskin [2]. It can be derived from the Kuhn-Tucker conditions or by elementary arguments based on the special structure of the problem. There are two important implications of this lemma:

(1) Marginal return must be above a threshold value, s, for a resource to be allocated

(2) For alternatives which receive allocation, $f_i'(x_i^*) = f_j'(x_j^*)$, i.e., at the optimum allocation point, the ratio $(\text{marginal return/marginal cost})_i$ is constant.

B. MODEL DESCRIPTION

As stated earlier, there are numerous considerations to be weighed in allocating strikes to arcs in the LOC network. From a mathematical viewpoint, it is assumed that the effectiveness of the network to its user may be measured in terms of how to obtain maximum flow at minimum cost. Therefore, the criterion to be used to allocate sorties is to minimize LOC effectiveness over time, taking into account the cost to repair damage on an arc. This is accomplished by allocating strikes to the set of arcs that maximize the costs to the user.

The effect of striking an arc is to reduce its capacity and increase the cost of a minimum cost circulation flow for a specified period of time. Therefore, in allocating N strikes, the selected arcs are the ones that maximize repair cost plus the product of the increase in minimum cost circulation flow and repair time. Specifically, if F is the cost of a minimum cost circulation flow before arc (i,j) is struck, F' is the cost of a minimum cost circulation flow after n_{ij} strikes on arc (i,j), and t_{ij} and k_{ij} are the repair time and repair cost, respectively, for damage caused by n_{ij} strikes, then the arcs selected for attack will be the ones that maximize:

$$\sum_{\text{all } (i,j)} (F' - F)\, t_{ij} + k_{ij}$$

$$\text{s.t.} \quad (1) \quad \sum_{\text{all } (i,j)} n_{ij} = N \; ; \; n_{ij} \geq 0$$

where: $$F = \sum_{\text{all } (i,j)} M_{ij} C_{ij}$$

$$F' = \sum_{\text{all } (i,j)} M_{ij}C_{ij} - d_{ij}(C_{ij} + C^u) \quad ; C_{ij} > 0$$
$$C^u << 0$$

Note: C^u is the unit flow cost on the universal arc.

$$d_{ij} = V_{ij}[1 - \exp(-VP_{ij}\ n_{ij})]$$

$$t_{ij} = (d_{ij}/V_{ij})\ T_{ij}$$

$$k_{ij} = (d_{ij}/V_{ij})\ K_{ij}$$

The objective function then becomes to maximize:

$$\sum_{\text{all}(i,j)} -T_{ij}V_{ij}(C_{ij} + C^u)\ [1 - \exp(-VP_{ij}\ n_{ij})]^2$$
$$+ K_{ij}[1 - \exp(-VP_{ij}\ n_{ij})]$$

This can be shown to be a continuous concave function of n_{ij} whose second derivative is negative. The constraint is convex; therefore, the above objective function becomes the $f_i(x_i)$ of Gibbs' Lemma.

To determine the allocation of N airstrikes, it is necessary to compute the value of the Lagrange Multiplier, s^*. By Gibbs' Lemma, it is known that there exists an $s^* > 0$ such that:

$$n_{ij} = 0 \text{ for } f_i'(x_i) < s^*$$

$$n_{ij} > 0 \text{ for } f_i'(x_i) \geq s^*$$

An iterative method for this computation is developed by Nugent [10].

The above procedure is used in Option (1) where the number of strikes is specified. For Option (2), a similar procedure is developed with the addition of another constraint. Recall that Option (2) requires a percentage by which the initial flow is to

be reduced. The additional constraint becomes:

$$(2) \quad \sum_{\text{all }(i,j)} M_{ij} \geq (1 - p)M'_{ij}$$

where: M'_{ij} is the initial network flow.

p is the percentage by which the initial flow is to be reduced.

Obviously, for the allocation of a maximum of N sorties, either constraint (1) or (2) will be active and the value of the Lagrange Multiplier, s^*, can be computed using Nugent's method.

If the vulnerability parameter of arc (i,j) takes cognizance of the probability of losing an aircraft on any mission against that arc, then the model seeks to interdict that set of arcs where the ratio $(\text{marginal return/marginal cost})_{ij}$ is constant.

C. COMPUTER SOLUTION PROCEDURE AND OUTPUT

The computer solution procedure operates in daily cycles, the strike planner specifying by Option (1) or (2) the specified or maximum number of strikes per day, respectively, and the number of days in the interdiction campaign. The computer solution differs from the optimal model described in B., in that strikes are allocated one at a time. Each time, the arc selected to be struck is the one which maximizes:

$$(F' - F)\, t_{ij} + k_{ij} \;.$$

The values F', F, t_{ij}, and k_{ij} are computed for one strike instead of for n_{ij} strikes. This is a marginal allocation procedure, the solution having the property that if an arc is selected for interdiction when N strikes are allocated, it will also be selected for

interdiction when more than N strikes are allocated. Since optimal allocation of strikes would not necessarily have this property, the selected arcs are optimal for one strike, but only approximate optimality for multiple strikes.

After each strike, the following information is output:

(1) Arc selected for strike
(2) Arc capacity before and after strike
(3) Actual arc flow before and after strike
(4) Day arc repair is to be completed
(5) Cumulative repair cost
(6) Network flow after strike
*(7) Network flow cost after strike
(8) Number of strikes against that arc for that day

*Note: Since the unit flow cost on the universal arc is an artificial cost used to convert a particular type of source-sink flow problem to one of circulation flow, it is not considered in the printed output calculation.

The end of a campaign day is determined by the option used. Using Option (1), the day terminates when N sorties have been allocated. Using Option (2), the day terminates when either N strikes have been allocated or the total network flow has been reduced to a specified percentage of the initial flow. The following information is output at the end of each day:

(1) Arcs selected for strike
(2) Number of strikes allocated against arc (i,j) for that day
(3) Number of strikes allocated against arc (i,j) for the campaign
(4) Total number of strikes allocated that day
(5) Total number of strikes allocated for the campaign

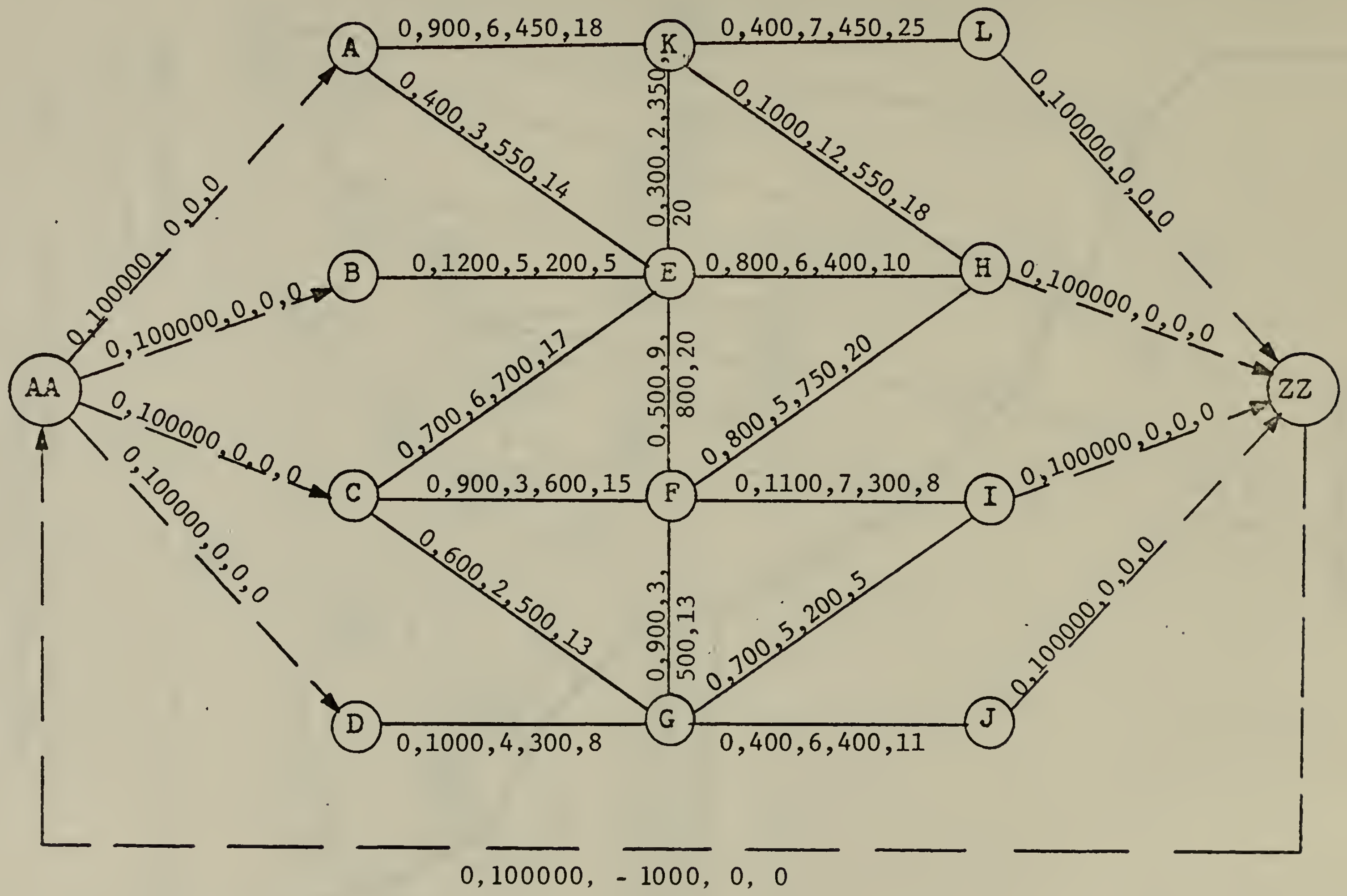
At the start of each day, all arc repair scheduled for that day has been completed. The resulting flow pattern is computed and arc flow, maximum network flow, and network flow cost are output.

V. SAMPLE PROBLEM

The network shown in Fig. 4 is used as a sample interdiction problem. It is the same network used previously but with the addition of maximum repair time and maximum repair cost parameters for each arc. Figure 5 depicts the same network, but node positions in relation to each other have been changed to indicate a more realistic lines-of-communication network. Artificial arcs from the super source are used to indicate that flow in the enemy sanctuary area is invulnerable to attack. Nodes A, B, C, and D are points on supply routes where arc capacity is subject to interdiction. The enemy objective is to move supplies from his sanctuary area to forward supply centers located at nodes L, H, I, and J. Nodes K, E, F, and G are route intersections. Figure 5 also gives the actual direction and magnitude of flow before the interdiction campaign begins.

The objective of the strike planner is to reduce the initial maximum network flow by 60 percent and thereafter, maintain this reduced flow for a campaign of 5 days duration. He has a maximum of 50 aircraft available each day to fly interdiction missions.

The computer solution to this problem is given in Appendix D.



L_{ij}, U_{ij}, C_{ij}, K_{ij}, T_{ij}

Figure 4. Network for Sample Problem

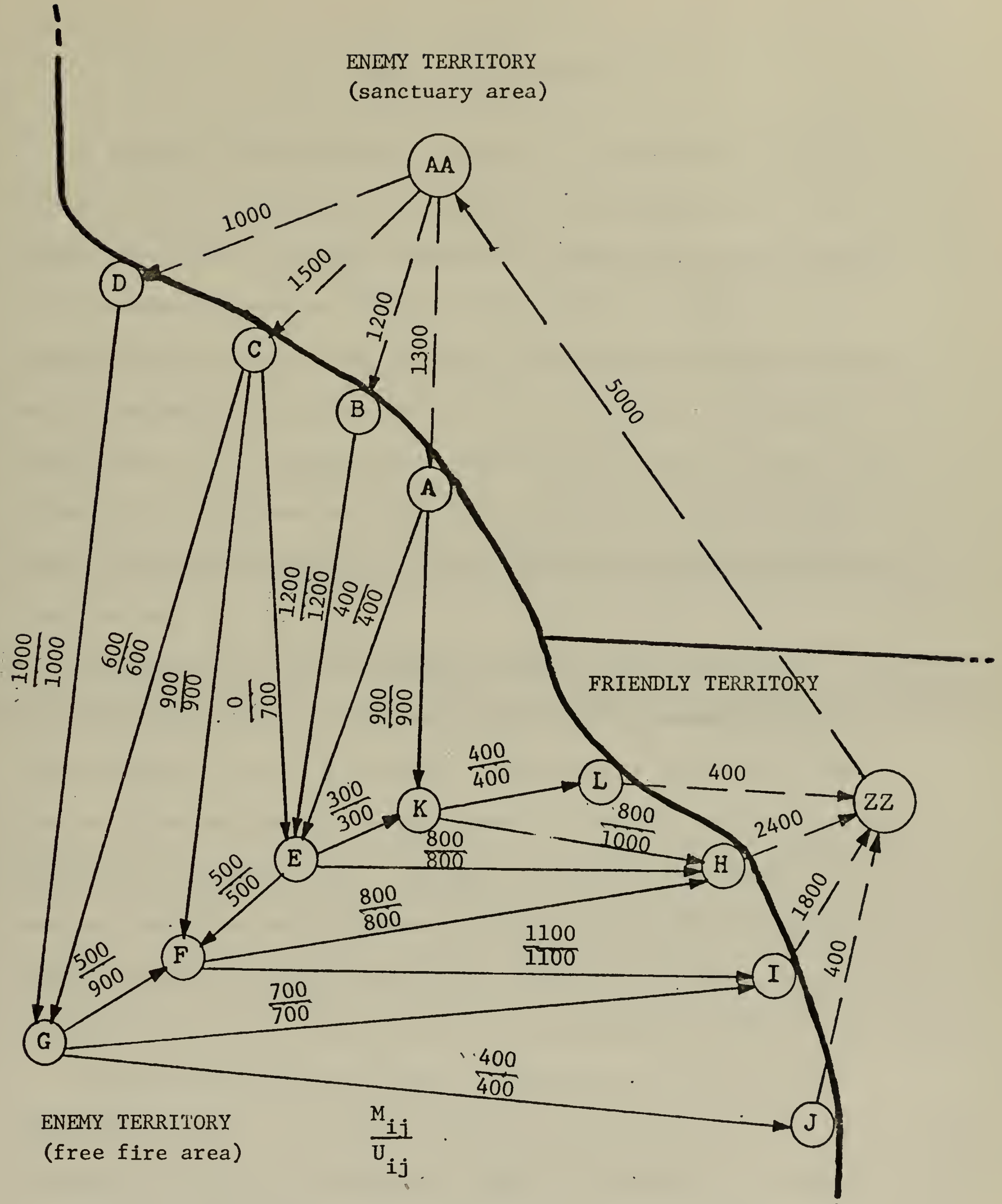


Figure 5. Network for Sample Problem Giving Direction and Magnitude of Initial Flow

VI. SUMMARY AND EXTENSIONS

A computer model has been presented for determining the daily allocation of airstrikes for interdicting a land lines-of-communication (LOC) network, assuming an exponential damage function. The allocation of sorties to arcs in the network is based on the assumptions that (1) flow of supplies is limited by network capacity, and (2) parameter values for each arc are independent. The model selects that arc to strike which maximizes the repair cost plus the product of the increase in minimum cost circulation flow and repair time. The procedure operates in daily cycles, performing repair on arcs that have been struck.

Two options are provided that allow the strike planner to (1) specify the number of strikes per day and the number of days in the campaign, or (2) prescribe a percentage by which the initial flow is to be reduced, the maximum number of strikes per day, and the number of campaign days. The upper and lower flow bounds, cost per unit flow, maximum repair time and cost, and the vulnerability parameter for each arc in the network are required as input data.

The model has been programmed in FORTRAN IV for use on the IBM 360/67 computer. Detailed information for each day in the campaign is listed in the computer output to include arcs selected for interdiction, network flow and network flow cost at the end of each day, and the number of sorties allocated to selected arcs.

There are almost an endless number of extensions that can be considered in the allocation of airstrikes to a LOC network. In the present model, it is assumed that only one type of aircraft is available to attack the network, i.e., one weapons system against y targets. A more practical and interesting approach is to assume that more than one type of aircraft is available for interdiction, each with a different kill probability against different types of targets, i.e., x weapons systems against y targets. The damage on an arc caused by n_{ij} strikes then becomes:

$$d_{ij} = V_{ij} \left[1 - \prod_{k=1}^{x} \exp(-VP_{ij,k}\, n_{ij,k})\right]$$

Another modification would be to vary the damage function itself. The exponential was used in this model because it exhibits decreasing marginal returns to scale and simplifies computational procedure; however, other such functions could be utilized. The exponential is a continuous approximation, for n_{ij} large and VP_{ij} small, of the function:

$$V_{ij}[1 - (1 - VP_{ij})^{n_{ij}}]$$

A contrast could be made of the damage caused by both distributions with actual damage data. Holliday [5] has done research in the field of damage assessment for the Advanced Research Projects Agency and his work might be used for such a contrast.

APPENDIX A

COMPUTER PROGRAM INPUT DATA

Card no.	Column no.	Format	Description
1	1-80	A	Program title. (PRONAM)
2	1-5	15	Maximum number of strikes that can be allocated each day. (N)
	6-10	15	The number of days in the campaign. (NUMDAY)
	11-15	15	The number of invulnerable or artificial arcs in the network. Include all arcs considered invulnerable to airstrike, artificial arcs connecting the dummy source to the true source(s), and artificial arcs connecting the true sink(s) to the dummy sink. (INVL)
	16-20	15	If equal to 1, specifies one-way flow on some arcs in the network. If equal to 2, specifies two-way flow on all arcs in the network. When arcs are two-way, a strike against one implies a strike against the other. (X2WAY)
	22-26	F4.2	Specifies the percentage of flow to be interdicted. If greater than 1.0, a fixed number of strikes per day (N) will be targeted. (REDU)
Arc Data Cards	1-4	A4	Name of arc. If two-way flow, must be the same for both arcs. (ARNAM)
	6-7	A2	Beginning node. (FROM)
	9-10	A2	Ending node. (TO)
	12-20	19	Lower bound on arc flow. (LBND)
	22-30	19	Arc capacity (upper bound on arc flow). (MCTY)
	32-40	19	Cost of one unit of flow on the arc. (JFCST)

	42-50	19	Maximum arc repair cost, i.e., the cost of repairing the arc from its lower bound capacity to its maximum capacity. (RMAXCT)
	52-60	19	Maximum arc repair time, i.e., the time required to repair the arc from its lower bound capacity to its maximum capacity. (MRTM)
	66-70	F5.3	Arc vulnerability parameter. (VP)
Last Card	1-2	A4	The word 'GO'.

Note: Arc data cards must be placed in the following order:

1. Universal arc.
2. Invulnerable and artifical arcs.
3. Vulnerable arcs.

APPENDIX B

GLOSSARY OF PROGRAM TERMS

ARNAM(k)	The designation of arc ij. (input data)
COSTMN(k)	Real number equivalent of MNCOST.
CRC	Cumulative repair cost for all interdicted arcs.
CRT(k)	Cumulative repair time for an interdicted arc.
CSTMNT	Real number equivalent of MNTCST.
DAMAGE(k)	Total reduction in capacity of arc ij caused by one or more strikes against it.
DAY	Counter for number of days up to NUMDAY.
FLOW(k)	Amount of flow on arc ij.
FROM(k)	Location of beginning node of arc ij. (input data)
GO	Flag used to denote last data card has been read.
IKEEP1	Used to save capacity of arc ij.
IKPOP2	Used to save capacity of the optimal arc to strike.
IKPOP1	If OPTARC is two-way, saves the capacity of the reverse direction arc.
IMNCST	Cost of the initial flow in the network.
IMNCUT	Cost of flow in the network after the optimal arc has been struck.
INFEAS	A flag used in the Out-of-Kilter subroutine to denote infeasible flow.
INVL	The number of invulnerable or artificial arcs in the network. (input data)
IOPARC	If an arc is two-way, specifies the reverse direction of the best arc to strike.
IRQFLO	Integer equivalent of REQFLO.

IS(k)	Counter for arcs that have been struck one or more times.
ISWTCH	Flag used to denote a two-way arc.
IT(k)	Counter for total strikes on arc ij during the campaign.
IV(k)	Counter for total strikes on arc ij during one day of the campaign.
JFCST(k)	Cost of one unit of flow on arc ij. (input data)
JFLOW(k)	Saves flow on OPTARC before strike. (for output)
J2WAY	Same as ISWTCH.
KFLOI	Saves flow on OPTARC after strike. (for output)
LBND(k)	Lower flow bound on arc ij. (input data)
LINE	Line counter used in the output subroutine.
MCAP(k)	Capacity of arc ij before next strike.
MCAPU	Saves the capacity of OPTARC before strike.
MCTY(k)	Flow capacity of arc ij. (input data)
MNCOST	Network flow cost before next strike.
MNTCST	Network flow cost after arc ij has been struck.
MRTM(k)	Maximum repair time for arc ij. (input data)
N	Maximum number of strikes that can be allocated during any one day. (input data)
NEWCAP(k)	Capacity of arc ij after strike.
NN(k)	Unique node number assigned by Subroutine Number.
NR	Counter for total strikes on all arcs during any one day.
NRC	Counter for total campaign strikes.
NUMARC	Maximum number of arcs in the network.
NUMDAY	Number of days in the campaign. (input data)

NUMNDE	Maximum number of nodes in the network.
OPTARC	The optimal arc to strike.
REDU	The percentage by which initial flow is to be reduced. (input data)
REPDAY(k)	Day on which arc ij is to be repaired.
RMAXCT(k)	Maximum repair cost for arc ij. (input data)
RPMCST(k)	Marginal repair cost for arc ij.
TME(k)	Marginal repair time for arc ij.
TO(k)	Location of ending node of arc ij. (input data)
TOTCST	Counter for cumulative campaign strikes.
TOTDST	Counter for cumulative day strikes.
VALUE(k)	The value of striking arc ij.
VP(k)	Vulnerability parameter of arc ij. (input data)
VULCAP(k)	The vulnerable capacity of arc ij.
XINCRE(k)	Marginal reduction in capacity on arc ij caused by an additional strike.
X2WAY	Input parameter that specifies whether or not all arcs are two-way.

APPENDIX C

PROGRAM LISTING

```
      DIMENSION XINCRE(150)
      COMMON/DAT01/INVL,DAY,NUMDAY,NUMARC,OPTARC,LINE,INFEAS
      COMMON/DAT02/S,NUMNDE,MCAPU,KFLCI,NR,N
      COMMON/DAT03/NRC,IMNCST,IMNCUT,X2WAY,MNCOST,MNTCST
      COMMON/DAT04/REPDAY(150),ARNAM(150),FROM(150),TO(150)
      COMMON/DAT05/RMAXCT(150),RPMCST(150),PI(150)
      COMMON/DAT06/LBND(150),NEWCP(150),MCAP(150),IS(150)
      COMMON/DAT07/MRTM(150),NN(150),MCTY(150),JFCST(150)
      COMMON/DAT08/FLOW(150),NL(150)
      COMMON/DAT09/VALUE(150),PRONAM(25),JFLOW(150)
      COMMON/DAT10/REQFLO,REDU,J2WY,ISWTCH,CRC
      COMMON/DAT11/IOPARC,TOTDST(150),TOTCST(150),IT(150)
      COMMON/DAT12/IV(150),DAMAGE(150),VP(150)
      COMMON/DAT13/IRQFLO,VULCAP(150),CRT(150)
      COMMON/DAT14/TME(150),VULCTY(150)
      INTEGER TOTDST,TOTCST,DAY,ARNAM,X2WAY,FROM,TO
      INTEGER FLOW,PI,OPTARC,REPDAY,S,RMAXCT,CRC
      DATA NAME/'GO'/

      OPTFLO = 0.
      REQFLO = 0.
      IRQFLO = 0
      I=0
C     READ PROGRAM TITLE AND PROGRAM PARAMETERS
      READ(5,505)(PRONAM(KK),KK=1,20)
      READ(5,500)N,NUMDAY,INVL,X2WAY,REDU
      WRITE(6,504)
      S = INVL + 2
C     READ ARC PARAMETERS
   10 I=I+1
      READ(5,510)ARNAM(I),FROM(I),TO(I),LBND(I),MCTY(I)
     1,JFCST(I),RMAXCT(I),MRTM(I),VP(I)
      IF(ARNAM(I) .EQ. NAME) GO TO 20
C     DETERMINE PARAMETERS FOR FIRST STRIKE
      VULCAP(I) = MCTY(I) - LBND(I)
      MCAP(I) = MCTY(I)
      DAMAGE(I) = VULCAP(I)*(1.0-EXP(-VP(I)))
      NEWCP(I) = FLOAT(MCTY(I))-DAMAGE(I) + .5
      RPMCST(I) = (FLOAT(RMAXCT(I))/VULCAP(I))*DAMAGE(I)
      TME(I) = (FLOAT(MRTM(I))/VULCAP(I))*DAMAGE(I)
      GO TO 10
   20 CONTINUE
      NUMARC = I-1
C     CALL NUMBER SUBROUTINE TO DESCRIBE NETWORK
      CALL NUMBER
C     INITIALIZE VARIABLES
      DO 30 K = 1,NUMARC
      IS(K) = 0
      IT(K) = 0
      IV(K) = 0
      FLOW(K) = 0
      NL(K) = 0
```

```
      REPDAY(K) = 0
      TOTCST(K) = 0
      TOTDST(K) = 0
      CRT(K) = 0.
      VALUE(K) =0.
      VULCTY(K) = 0.
      XINCRE(K) = 0.
   30 CONTINUE
C     SOLVE UNINTERDICTED NETWORK
      CALL OCK(NUMNDE,NUMARC,FROM,TO,JFCST,MCAP,LBND,FLOW,
     1PI,NL,INFEAS)
      IF(INFEAS.EQ.1) GO TO 70
      OPTFLO = FLOAT(FLOW(1))
      REQFLO = OPTFLO*(1.-REDU)
      IRQFLO = REQFLO
C     PRINT PROGRAM TITLE AND MODE OF OPERATION
      CALL OUTPUT(1)
      DAY = 0
      CALL OUTPUT(6)
C     FIND MAX FLOW,FLOW COST,AND PRINT RESULTS
      CALL FLOCST(FLOW,JFCST,NUMARC,IMNCST,INVL,1)
      CALL OUTPUT(7)
      DAY = 1
      NR = 0
      NRC = 0
      CRC = 0
C     INTERDICTION OF NETWORK BEGINS HERE
   40 NR = NR + 1
      NRC = NRC + 1
      INFEAS = 0
      DO 45 I=S,NUMARC
      JFLOW(I) = FLOW(I)
   45 CONTINUE
C     CALL STKARC TO DETERMINE BEST ARC TO STRIKE
      CALL STKARC
      IF(INFEAS.EQ.1) GO TO 70
C     OPTARC IS THE BEST ARC
      MCAPU = MCAP(OPTARC)
      KFLOI = FLOW(OPTARC)
      ISAVE = OPTARC
      K = IOPARC
C     STORE COST OF REPAIR AND COST OF STRIKE
      CRC = CRC + RPMCST(OPTARC) + .5
   50 CONTINUE
      CRT(K) = CRT(K) + TME(K)
      REPDAY(K) = DAY + CRT(K) + .5
C     COUNTER FOR ARCS THAT HAVE BEEN STRUCK
      IS(K) = IS(K) + 1
C     COUNTER FOR DAY STRIKES
      IV(K) = IV(K) + 1
C     COUNTER FOR CAMPAIGN STRIKES
      IT(K) = IT(K) + 1
```

```
      TOTDST(K) =  IV(K)
      TOTCST(K) = IT(K)
C
C     DETERMINE PARAMETERS FOR NEXT STRIKE
C
      J = IS(K) + 1
      MCAP(K) = NEWCP(K)
      VULCTY(K)=MCAP(K) - LBND(K)
      XINCRE(K)=VULCTY(K)*(1.-EXP(-VP(K)))
      NEWCP(K)=FLOAT(MCAP(K))-XINCRE(K) + .5
      DAMAGE(K) = MCTY(K) - NEWCP(K)
      RPMCST(K) = (FLOAT(RMAXCT(K))/VULCAP(K))*XINCRE(K)
      TME(K) =(FLOAT(MRTM(K))/VULCAP(K))*XINCRE(K)
C
C     IF ARC IS 2-WAY, GO BACK AND COMPUTE
C     PARAMETERS FOR RETURN ARC
C
      IF (ISWTCH.EQ.1) GO TO 60
      K = ISAVE
      ISWTCH = 1
      GO TO 50
   60 CONTINUE
C
C     PRINT STRIKE RESULTS
C
      CALL OUTPUT(2)
C
   70 CONTINUE
      IF(INFEAS .EQ. 1) GO TO 130
      IF(FLOW(1) .LT. LBND(1)) GO TO 130
      IF(FLOW(1) .EQ. 0) GO TO 140
C
C     IF REDU IS < 1,NUMBER OF STRIKES IS NOT
C     CONSTANT. IF NUMBER OF STRIKES = NUMBER
C     SPECIFIED IN INPUT OR IF CAPACITY
C     HAS BEEN REDUCED TO SPECIFIED AMOUNT,
C     DAY IS DONE.
C
      IF(REDU.LE.1.0)GO TO 80
      IF(NR .EQ. N) GO TO 90
      GO TO 40
   80 CONTINUE
      IF(FLOW(1).LE.REQFLO)GO TO 90
      IF(NR.EQ.N)GO TO 90
      GO TO 40
   90 CALL OUTPUT(5)
      IF(DAY .EQ. NUMDAY) GO TO 150
      LINE = 48
C
C     RE-INITIALIZE PARAMETERS FOR NEXT DAY
C
  100 DAY = DAY + 1
      NR =0
      DO 110 M=S,NUMARC
      IV(M) = 0
      TOTDST(M) = 0
  110 CONTINUE
C
C     PERFORM REPAIR ON ARCS
C
      DO 120 M = S,NUMARC
      IF(IS(M) .EQ. 0) GO TO 120
      MCAP(M)=FLOAT(MCAP(M))+(1./CRT(M))*DAMAGE(M)
      VULCTY(M)=MCAP(M) - LBND(M)
      XINCRE(M)=VULCTY(M)*(1.-EXP(-VP(M)))
      NEWCP(M)=FLOAT(MCAP(M)) - XINCRE(M) + .5
      DAMAGE(M)=MCTY(M) - NEWCP(M)
      RPMCST(M) = (FLOAT(RMAXCT(M))/VULCAP(M))*XINCRE(M)
      TME(M)=(FLOAT(MRTM(M))/VULCAP(M))*XINCRE(M)
      CRT(M) = CRT(M) - 1.
  120 CONTINUE
```

```
C     SOLVE NETWORK FOR NEXT DAY

      CALL OOK(NUMNDE,NUMARC,FROM,TO,JFCST,MCAP,LBND,FLOW,
     1PI,NL,INFEAS)
      IF(INFEAS .EQ. 1) GO TO 70

C     FIND  FLOW COST

      CALL FLOCST(FLOW,JFCST,NUMARC,IMNCST,INVL,1)

C     PRINT NETWORK FLOW PATTERN AND ITS COST

      CALL OUTPUT(6)
      CALL OUTPUT(7)
      IF(FLOW(1).GT.REQFLO)GO TO 40
      GO TO 100
  130 CALL OUTPUT(3)
      GO TO 90
  140 CALL OUTPUT(4)
      GO TO 90
  150 CONTINUE
      CALL EXIT


  500 FORMAT(4I5,1X,F4.2)
  504 FORMAT(1H1,10X)
  505 FORMAT(20A4)
  510 FORMAT(A4,2(1X,A2),5(1X,I9),5X,F5.3)
      END
```

```
C     OOK IS THE FULKERSON OUT OF KILTER ALGORITHM


      SUBROUTINE OOK(NODES,ARCS,I,J,COST,HI,LO,FLOW,PI
     1,NA,INFEAS)
      DIMENSION I(150),J(150),COST(150),HI(150),
     1LO(150),FLOW(150)
      DIMENSION PI(150),NA(150)
      INTEGER NODES,ARCS,I,J,COST,HI,LO,FLOW,PI,NA,INFEAS
      INTEGER A,AA,N,SRC,SNK,DEL,INF,C,AOK,COK,N1,N2,
     1INC,LABEL
C         DEFINITION OF CALLING SEQUENCE
C
C         NAME      USE
C
C         NODES     NUMBER OF NODES
C         ARCS      NUMBER OF ARCS
C         I         LIST OF FROM (BEGINNING) NODES
C         J         LIST OF TO (ENDING) NODES
C         COST      UNIT COST OF FLOW ON ARCS
C         HI        UPPER BOUNDS FOR ARCS
C         LO        LOWER BOUNDS FOR ARCS
C         FLOW      AMOUNT OF FLOW IN ARCS
C         PI        NODE PRICES
C         NA        SCRATCH LIST FOR NODES LABELING
C         INFEAS    FLAG DENOTING THE CONDITION OF OUTPUT

      INF = 2147483647
      AOK = 0

C     LOOK FOR AN OUT OF KILTER ARC

      AA=1
  100 N1 = I(AA)
      N2 = J(AA)
      C = COST(AA) + PI(N1) - PI(N2)
   40 IF(FLOW(AA).LT.LO(AA).OR.(C.LT.0.AND.FLOW(AA).LT.
     1HI(AA))) GO TO 50
      IF(FLOW(AA).GT.HI(AA).OR.(C.GT.0.AND.FLOW(AA).GT.
     1LO(AA))) GO TO 60
   90 AA = AA + 1
      IF(AA.LE.ARCS)GO TO 100

C     NO OUT OF KILTER ARCS LEFT

      INFEAS = 0
      RETURN

C     OUT OF KILTER ARC FOUND

   50 SRC = J(AA)
      SNK = I(AA)
      LABEL= +AA
      GO TO 200
   60 SRC = I(AA)
      SNK = J(AA)
      LABEL = -AA

C     SAVE LABELS IF LAST OPERATION WAS INCREASING NODE
C     PRICES ON THIS ARC

  200 IF(AA.EQ.AOK.AND.NA(SRC).NE.0) GO TO 205
      DO 201 N =1,NODES
      NA(N) = 0
  201 CONTINUE
      AOK = AA
  205 COK = C
      NA(SRC) = LABEL
```

```
C     LABEL

  210 LABEL = 0
      DO 250 A =1,ARCS
      N1 = I(A)
      IF(N1.LT.0) GO TO 250
      N2 = J(A)
      IF(NA(N1).EQ.0.AND.NA(N2).EQ.0) GO TO 250
      IF(NA(N1).NE.0.AND.NA(N2).NE.0)GO TO 245
      C = COST(A) + PI(N1) -PI(N2)
      IF(NA(N1).EQ.0) GO TO 220
      IF(FLOW(A).GE.HI(A).OR.(FLOW(A).GE.LO(A).AND.C.GT.0))
     1GO TO 245
      NA(N2) = A
      GO TO 240
  220 IF(FLOW(A).LE.LO(A).OR.(FLOW(A).LE.HI(A).AND.C.LT.0))
     1GO TO 245
      NA(N1) = -A
  240 LABEL = 1

C     NODE LABELLED, TEST FOR BREAKTHRU

      IF(NA(SNK).NE.0) GO TO 260
  245 I(A) = -N1
  250 CONTINUE

C     GO BACK AND DO MORE LABELLING IF SOME NODE WAS
C     LABELLED ON LAST PASS

      IF(LABEL.NE.0) GO TO 210

C     RESTORE POSITIVE SIGNS TO FIRST NODE LIST

  260 DO 270 A=1,ARCS
      I(A) = IABS(I(A))
  270 CONTINUE

C     IF NO LABELLING DONE ON LAST PASS, GO TO
C     INCREASE PIE

      IF(LABEL.EQ.0) GO TO 400

C     BREAKTHRU,  FIND THE INCREMENT

  300 INC = INF

C     FOLLOW PATH BACK FROM SOURCE

      N = SRC
  310 A = IABS(NA(N))
      IF(NA(N).LT.0) GO TO 315
      N2 = I(A)
      C = COST(A) - PI(N) + PI(N2)
      IF(C.GT.0)INC = MIN0(INC,LO(A) - FLOW(A))
      IF(C.LE.0)INC = MIN0(INC,HI(A) -FLOW(A))
      GO TO 340
  315 N2 = J(A)
      C = COST(A) + PI(N) -PI(N2)
  320 IF(C.LT.0)INC = MIN0(INC,FLOW(A) -HI(A))
      IF(C.GE.0)INC = MIN0(INC,FLOW(A) -LO(A))
  340 N = N2
      IF(N.NE.SRC)GO TO 310

C     INCREMENTS

  350 A = IABS(NA(N))
      IF(NA(N).LT.0)GO TO 360
      FLOW(A) = FLOW(A) + INC
      N = I(A)
      GO TO 370
  360 FLOW(A) = FLOW(A) -INC
      N = J(A)
```

```
  370 IF(N.NE.SRC)GO TO 350

C     FLOW INCREMENTED, RETURN TO KILTER TEST

      NA(N) = 0
      GO TO 100

C     CHANGE PI

  400 DEL = INF

C     FIND INCREMENT

      DO 420 A=1,ARCS
      N1 = I(A)
      N2 = J(A)
      IF(NA(N1).EQ.0.AND.NA(N2).EQ.0)GO TO 420
      IF(NA(N1).NE.0.AND.NA(N2).NE.0)GO TO 420
      C = COST(A) + PI(N1) - PI(N2)
      IF(NA(N2).EQ.0.AND.FLOW(A).LT.HI(A)) DEL =MIN0(DEL,C)
      IF(NA(N2).NE.0.AND.FLOW(A).GT.LO(A)) DEL =MIN0(DEL,-C)
  420 CONTINUE
      IF(DEL.NE.INF) GO TO 430
      IF(FLOW(AA).EQ.LO(AA).OR.FLOW(AA).EQ.HI(AA))GO TO 425

C     INFEASIBLE SOLUTION

      INFEAS = 1
      RETURN

C     INCREASE PI

  425 DEL = IABS(COK)
  430 DO 450 N =1,NODES
      IF(NA(N).EQ.0) PI(N) = PI(N) + DEL
  450 CONTINUE

C     GO BACK TO KILTER TEST

      GO TO 100
      END
```

```
      SUBROUTINE FLOCST(FLOW,COST,NUMARC,MINCT,INVL,JUNIV)
      DIMENSION FLOW(150),COST(150)
      INTEGER FLOW,COST

      MINCT = 0
      IUNV = INVL

C     IF JUNIV = 0, USE COST OF UNIVERSAL ARC

      IF(JUNIV .EQ. 0) IUNV = 1
      DO 10 I = IUNV,NUMARC
   10 MINCT = MINCT + FLOW(I)*COST(I)
      RETURN
      END



      SUBROUTINE NUMBER
      COMMON/DAT01/INVL,DAY,NUMDAY,NUMARC,OPTARC,LINE,INFEAS
      COMMON/DAT02/S,NUMNDE,MCAPU,KFLOI,NR,N
      COMMON/DAT04/REPDAY(150),ARNAM(150),FROM(150),TO(150)
      COMMON/DAT07/MRTM(150),NN(150),MCTY(150),JFCST(150)
      INTEGER TOTDST,TOTCST,DAY,ARNAM,X2WAY,FROM,TO

C     NUMBER THE NODES

      NUMNDE = 0
      DO 30 L=1,NUMARC
      NN(NUMNDE + 1) =FROM(L)
      FROM(L) = NODNUM(FROM(L),NN)
      NUMNDE =MAX0(NUMNDE,FROM(L))
   30 CONTINUE
      DO 40 L =1,NUMARC
      NN(NUMNDE + 1) = TO(L)
      TO(L) = NODNUM(TO(L),NN)
      NUMNDE = MAX0(NUMNDE,TO(L))
   40 CONTINUE

C     CHECK FOR DEAD NODES

      DO 48 I =1,NUMNDE
      DO 42 L =1,NUMARC
      IF(FROM(L).EQ.1)GO TO 44
   42 CONTINUE
      WRITE(6,522)NN(I)
   44 CONTINUE
      DO 46 L =1,NUMARC
      IF(TO(L).EQ.1)GO TO 48
   46 CONTINUE
      WRITE(6,530)NN(I)
   48 CONTINUE
      RETURN

  522 FORMAT(1H1, 22HNO ARC BEGINS AT NODE , A4/1H1)
  530 FORMAT(1H1, 22HNO ARC ENDS AT NODE   , A4/1H1)
      END



      FUNCTION NODNUM(III,NN)
      DIMENSION NN(1)
      COMMON/DAT02/S,NUMNDE,MCAPU,KFLOI,NR,N

      NODNUM = NUMNDE + 1
      IF(NUMNDE.EQ.0)RETURN
      DO 10 I=1,NUMNDE
      IF(III.EQ.NN(I)) NODNUM = I
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE STKARC
      COMMON/DAT01/INVL,DAY,NUMDAY,NUMARC,OPTARC,LINE,INFEAS
      COMMON/DAT02/S,NUMNDE,MCAPU,KFLCI,NR,N
      COMMON/DAT03/NRC,IMNCST,IMNCUT,X2WAY,MNCOST,MNTCST
      COMMON/DAT04/REPDAY(150),ARNAM(150),FROM(150),TO(150)
      COMMON/DAT05/RMAXCT(150),RPMCST(150),PI(150)
      COMMON/DAT06/LBND(150),NEWCP(150),MCAP(150),IS(150)
      COMMON/DAT07/MRTM(150),NN(150),MCTY(150),JFCST(150)
      COMMON/DAT08/FLOW(150),NL(150)
      COMMON/DAT09/VALUE(150),PRONAM(25),JFLOW(150)
      COMMON/DAT10/REQFLO,REDU,J2WY,ISWTCH,CRC
      COMMON/DAT11/IOPARC,TOTDST(150),TOTCST(150),IT(150)
      COMMON/DAT12/IV(150),DAMAGE(150),VP(150)
      COMMON/DAT13/IRQFLO,VULCAP(150),CRT(150)
      COMMON/DAT14/TME(150),VULCTY(150)
      INTEGER TOTDST,TOTCST,DAY,ARNAM,X2WAY,FROM,TO
      INTEGER FLOW,PI,OPTARC,REPDAY,S,RMAXCT,CRC

C     INITIALIZE PARAMETERS

      MNTCST = 0
      MNCOST = 0
      CSTMNT=0.
      COSTMN=0.
      UPVAL = -10000000.

C     SOLVE FOR LAST FLOW PATTERN

      CALL OOK(NUMNDE,NUMARC,FROM,TO,JFCST,MCAP,LBND,FLOW,
     1PI,NL,INFEAS)
      IF(INFEAS.EQ.1)RETURN
      CALL FLOCST(FLOW,JFCST,NUMARC,MNCOST,INVL,0)
      DO 100 I=S,NUMARC
      IF(FLOW(I).EQ.0)GO TO 100
      IKEEP1=MCAP(I)
      MCAP(I)=NEWCP(I)

C     SOLVE INTERDICTED FLOW PATTERN FOR
C     SELECTED ARCS AND FIND FLOW COST

      CALL OOK(NUMNDE,NUMARC,FROM,TO,JFCST,MCAP,LBND,FLOW,
     1PI,NL,INFEAS)
      IF(INFEAS.EQ.1)RETURN
      CALL FLOCST(FLOW,JFCST,NUMARC,MNTCST,INVL,0)

C     FIND VALUE OF STRIKING ARCS AND
C     SAVE MAXIMUM VALUE

      MCAP(I)=IKEEP1
      CSTMNT=MNTCST
      COSTMN=MNCOST
      VALUE(I)=(CSTMNT-COSTMN)*TME(I)+RPMCST(I)
      IF(VALUE(I).LT.UPVAL)GO TO 90
      UPVAL=VALUE(I)

C     OPTARC IS THE BEST ARC TO STRIKE

      OPTARC=I
   90 CONTINUE
  100 CONTINUE
      DO 52 I=S,NUMARC
      IF(FROM(OPTARC).EQ.TO(I).AND.TO(OPTARC).EQ.FROM(I)
     1.AND.ARNAM(OPTARC).EQ.ARNAM(I))GO TO 54
   52 CONTINUE
   53 CONTINUE
      J2WY = 1

C     IOPARC IS THE REVERSE DIRECTION ARC

      IOPARC = OPTARC
```

```
      GO TO 56
   54 J2WY = 2
      IOPARC = I
      ISWTCH = J2WY
      IF(X2WAY.EQ.1) GO TO 53
   56 CONTINUE
      IKPOP2 = MCAP(IOPARC)
      IKPOP1 = MCAP(OPTARC)
      MCAP(OPTARC) = NEWCP(OPTARC)
      MCAP(IOPARC) = NEWCP(IOPARC)

C     STRIKE BEST ARC AND FIND FLOW COST

      CALL OCK(NUMNDE,NUMARC,FROM,TO,JFCST,MCAP,LBND,FLOW,
     1PI,NL,INFEAS)
      IF(INFEAS.EQ.1)RETURN
      CALL FLOCST(FLOW,JFCST,NUMARC,IMNCUT,INVL,1)
      MCAP(OPTARC) = IKPOP1
      MCAP(IOPARC) = IKPOP2
      RETURN
      END
```

```
      SUBROUTINE OUTPUT(J)
      COMMON/DAT01/INVL,DAY,NUMDAY,NUMARC,OPTARC,LINE,INFEAS
      COMMON/DAT02/S,NUMNDE,MCAPU,KFLOI,NR,N
      COMMON/DAT03/NRC,IMNCST,IMNCUT,X2WAY,MNCOST,MNTCST
      COMMON/DAT04/REPDAY(150),ARNAM(150),FROM(150),TO(150)
      COMMON/DAT05/RMAXCT(150),RPMCST(150),PI(150)
      COMMON/DAT06/LBND(150),NEWCP(150),MCAP(150),IS(150)
      COMMON/DAT07/MRTM(150),NN(150),MCTY(150),JFCST(150)
      COMMON/DAT08/FLOW(150),NL(150)
      COMMON/DAT09/VALUE(150),PRONAM(25),JFLOW(150)
      COMMON/DAT10/REQFLO,REDU,J2WY,ISWTCH,CRC
      COMMON/DAT11/IOPARC,TOTDST(150),TOTCST(150),IT(150)
      COMMON/DAT12/IV(150),DAMAGE(150),VP(150)
      COMMON/DAT13/IRQFLO,VULCAP(150),CRT(150)
      INTEGER TOTDST,TOTCST,DAY,ARNAM,X2WAY,FROM,TO
      INTEGER FLOW,PI,OPTARC,REPDAY,S,RMAXCT,CRC

      IF(J.GT.1) GO TO  20
      WRITE(6,506)(PRONAM(I),I=1,20)
      IF(REDU.LT.1.)GO TO 10
      WRITE(6,507)NUMDAY,N
      LINE = 8
      RETURN
   10 WRITE(6,510)N,NUMDAY,FLOW(1),IRQFLO,IRQFLO,N
      WRITE(6,504)
      LINE = 0
      RETURN
   20 IF(LINE.NE.48) GO TO  30
      WRITE(6,504)
      LINE = 0
   30 IF(LINE.NE.0)GO TO  40
   40 GO TO(100,100,150,160,210,50,180),J
   50 IARC = 0
      IF(DAY.EQ.0)WRITE(6,502)
      IF(DAY.EQ.0)GO TO 70
      WRITE(6,519)DAY
   60 LINE = 0
   70 WRITE(6,508)
   80 IARC = IARC + 1
      IF(IARC.GT.NUMARC) GO TO 90
      IF(FLOW(IARC).EQ.0.AND.DAY.NE.0) GO TO 80
      LINE = LINE + 1
      II = FROM(IARC)
      IJ = TO(IARC)
      WRITE(6,509)ARNAM(IARC),NN(II),NN(IJ),LBND(IARC),
     1MCAP(IARC),JFCST(IARC),RMAXCT(IARC),MRTM(IARC),
     2FLOW(IARC),VP(IARC)
      IF(LINE.NE.49) GO TO 80
      WRITE(6,504)
      GO TO 60
   90 CONTINUE
      LINE = 0
      RETURN
  100 CONTINUE
      IF(LINE +6.GE.48) GO TO 200
      IF(LINE.NE.0) GO TO 120
  110 CONTINUE
      WRITE(6,503)DAY
      WRITE(6,500)
      LINE = LINE + 4
  120 CONTINUE
      K = OPTARC
      III=FROM(K)
      JJJ=TO(K)
      WRITE(6,501)NR,NN(III),NN(JJJ),MCAPU,MCAP(K),
     1JFLOW(K),KFLOI,REPDAY(K),CRC,FLOW(1),IMNCUT
     2,IS(K)
  130 LINE = LINE + 1
      RETURN
  150 WRITE(6,513)DAY
```

```
      LINE =LINE + 8
      RETURN
  160 WRITE(6,512)DAY
      LINE = LINE + 8
      RETURN
  180 WRITE(6,515) IMNCST,FLOW(1)
      IF(DAY.EQ.0)GO TO 190
      IF(FLOW(1).GT.REQFLO)GO TO 190
      WRITE(6,516)DAY
  190 CONTINUE
      WRITE(6,504)
      LINE = 0
      RETURN
  200 LINE =48
      GO TO  20
  210 CONTINUE
      WRITE(6,520)DAY
      WRITE(6,521)DAY
      DO 220 JARC=S,NUMARC
      KK = FROM(JARC)
      KJ = TO(JARC)
      IF(TOTDST(JARC).EQ.0.AND.TOTCST(JARC).EQ.0)
     1 GO TO 220
      IF(FLOW(JARC).EQ.0)GO TO 220
      WRITE(6,522)ARNAM(JARC),NN(KK),NN(KJ),TOTDST(JARC),
     1TOTCST(JARC)
  220 CONTINUE
      WRITE(6,523)NR,NRC
      RETURN

  500 FORMAT(T30,'ARC CAPACITY',T48,'ARC FLOW',T61,'TO BE',
     1T71,'CUM.',T77,'MAXIMUM',T87,'NETWORK',T96,'NUMBER',/
     2,T10,'MISSICN',T28,'BEFORE  AFTER',4X,'BEFORE  AFTER'
     3,2X,'REPAIRED',2X,'REPAIR',1X,'NETWORK',4X,'FLOW',4X,
     4'OF ARC',/,T10,'NUMBER',2X,'FROM  TO',2X,'STRIKE',2X,
     5'STRIKE',3X,'STRIKE  STRIKE',2X,'ON DAY',4X,'COST',3X,
     6'FLOW',6X,'COST',4X,'STRIKES')

  501 FORMAT(10X,I3,5X,A2,3X,A2,2X,2(I5,3X),1X,2(I5,3X),
     1I4,5X,I5,1X,I7,1X,I9,3X,I4)

  502 FORMAT('0',19X,'INITIAL FLOW PATTERN FOR LOC NETWORK'
     1,/)

  503 FORMAT('0',8X,'STRIKES TARGETED FOR DAY',I3,'.')

  504 FORMAT(1H1,10X)

  506 FORMAT(1H ,28X,20A4,///)

  507 FORMAT(1H ,32X, 23HCAMPAIGN WILL  LAST FOR   ,I3,
     112H DAYS, WITH  ,I3,' MISSIONS PER DAY.')

  508 FORMAT(44X,'LOWER',6X,'UPPER',13X,'MAX.',5X,'MAX.',15X
     1,'ARC',/,20X,'ARC',4X,'FROM',5X,'TO',6X,'FLOW',7X,
     2'FLOW',5X,'FLOW',4X,'REPAIR',3X,'REPAIR',4X,'ARC',7X,
     3'VULNER.',/,20X,'NAME',3X,'NODE',4X,'NODE',5X,'BOUND',
     46X,'BOUND',4X,'COST',5X,'COST',5X,'TIME',5X,'FLOW',
     56X,'PARAM.')

  509 FORMAT(2H  ,18X,A4,4X,A2,6X,A2,1X,I8,5X,I8,1X,I7,
     12(4X,I5),1X,I8,6X,F5.3)

  510 FORMAT(36X,'MAXIMUM NUMBER OF STRIKES EACH DAY :'
     1,I3,/,36X,'NUMBER OF DAYS IN THE CAMPAIGN     :',I3,
     2//,36X,'NOTE: MISSIONS WILL CONTINUE EACH DAY UNTIL',
     3/,36X,'ONE OF THE FOLLOWING OCCURS :',/,36X,
     4'(1) UNINTERDICTED FLOW (',I4,' UNITS) IS',/,40X,
     5'REDUCED TO ',I4,' UNITS.',/,36X,'(2) FLOW AT THE ',
     6'BEGINNING OF EACH',/,40X,'DAY IS REDUCED TO ',I4,
     7' UNITS.',/,36X,'(3) ',I3,' STRIKES HAVE BEEN '
     8,'TARGETED.')
```

```
512 FORMAT(1H ,///10X,27HFLOW TOTALLY STOPPED ON DAY   ,
     1I10,///)
513 FORMAT(1H ,///10X,30HMINIMUM THRU-PUT CANNOT BE MET,
     1' ON DAY',I10,///)
515 FORMAT('0',//,20X,'FLOW COST  :',I8,/,20X,
     1'THROUGHPUT :',I8)
516 FORMAT('0',//,20X,'NO STRIKES REQUIRED ON DAY',I3,'.')
519 FORMAT('0',19X,'ALL ARC REPAIR SCHEDULED FOR DAY',I3,
     1' HAS BEEN COMPLETED. ',/,20X,'RESULTING FLOW PATTERN'
     2,' IS AS FOLLOWS:',/)
520 FORMAT('0',/,20X,'RECAP FOR DAY',I3,':',/,20X,
     1'*****************')
521 FORMAT('0',45X,'TOTAL NUMBER    TOTAL NUMBER',/,21X,
     1' ARC',21X,'ARC STRIKES     ARC STRIKES',/,22X,'NAME',
     24X,'FROM',5X,'TO',5X,'FOR DAY',I3,6X,'FOR CAMPAIGN')
522 FORMAT(2H  ,20X,A4,5X,A2,6X,A2,9X,I4,12X,I4)
523 FORMAT('0',/,22X,'NUMBER STRIKES THIS DAY      :',
     1I9,/,22X,'NUMBER STRIKES THIS CAMPAIGN :',I9)
    END
```

APPENDIX D

COMPUTER OUTPUT FOR SAMPLE PROBLEM

```
INTERDICTION OF A LINES OF COMMUNICATION NETWORK

   MAXIMUM NUMBER OF STRIKES EACH DAY : 50
   NUMBER OF DAYS IN THE CAMPAIGN     :  5

   NOTE: MISSIONS WILL CONTINUE EACH DAY UNTIL
   ONE OF THE FOLLOWING OCCURS :
   (1) UNINTERDICTED FLOW (5000 UNITS) IS
       REDUCED TO 2000 UNITS.
   (2) FLOW AT THE BEGINNING OF EACH
       DAY IS REDUCED TO 2000 UNITS.
   (3)  50 STRIKES HAVE BEEN TARGETED.
```

```
INITIAL FLOW PATTERN FOR LOC NETWORK

                                      LOWER          UPPER                     MAX.        MAX.                     ARC
ARC       FROM        TO              FLOW           FLOW          FLOW      REPAIR      REPAIR        ARC          VULNER.
NAME      NODE        NODE            BOUND          BOUND         COST       COST        TIME         FLOW         PARAM.
ZZAA       ZZ          AA               0           100000        -1000          0           0          5000         0.0
AAA        AA          A                0           100000            0          0           0          1300         0.0
AAB        AA          B                0           100000            0          0           0          1200         0.0
AAC        AA          C                0           100000            0          0           0          1500         0.0
AAD        AA          D                0           100000            0          0           0          1000         0.0
H  ZZ      H           ZZ               0           100000            0          0           0          2400         0.0
I  ZZ      I           ZZ               0           100000            0          0           0          1800         0.0
J  ZZ      J           ZZ               0           100000            0          0           0           400         0.0
L  ZZ      L           ZZ               0           100000            0          0           0           400         0.0
B  E       B           E                0             1200            5        200           5          1200         0.070
B  E       E           B                0             1200            5        200           5             0         0.070
C  E       C           E                0              700            6        700          17             0         0.020
C  E       E           C                0              700            6        700          17             0         0.020
C  F       C           F                0              900            3        600          15           900         0.060
C  F       F           C                0              900            3        500          15             0         0.060
C  G       C           G                0              600            2        500          13           600         0.030
C  G       G           C                0              600            2        500          13             0         0.030
D  G       D           G                0             1000            4        300           8          1000         0.050
D  G       G           D                0             1000            4        300           8             0         0.050
E  H       E           H                0              800            6        400          10           800         0.060
E  H       H           E                0              800            6        400          10             0         0.060
E  F       E           F                0              500            9        800          20           500         0.020
E  F       F           E                0              500            9        800          20             0         0.020
F  G       F           G                0              900            3        500          13             0         0.010
F  G       G           F                0              900            3        500          13           500         0.010
F  H       F           H                0              800            5        750          20           800         0.030
F  H       H           F                0              800            5        750          20             0         0.030
F  I       F           I                0             1100            7        300           8          1100         0.040
F  I       I           F                0             1100            7        300           8             0         0.040
G  I       G           I                0              700            5        200           5           700         0.050
G  I       I           G                0              700            5        200           5             0         0.050
G  J       G           J                0              400            6        400          11           400         0.050
G  J       J           G                0              400            6        400          11             0         0.050
A  K       A           K                0              900            6        450          18           900         0.090
A  K       K           A                0              900            6        450          18             0         0.090
A  E       A           E                0              400            3        550          14           400         0.050
A  E       E           A                0              400            3        550          14             0         0.050
K  E       K           E                0              300            2        350          20             0         0.030
K  E       E           K                0              300            2        350          20           300         0.030
K  L       K           L                0              400            7        450          25           400         0.030
K  L       L           K                0              400            7        450          25             0         0.030
K  H       K           H                0             1000           12        550          18           800         0.010
K  H       H           K                0             1000           12        550          18             0         0.010


FLOW COST   :     61900
THROUGHPUT  :      5000
```

```
STRIKES TARGETED FOR DAY  1.
                          ARC  CAPACITY          ARC  FLOW        TO BE        CUM.   MAXIMUM      NETWORK   NUMBER
MISSION                 BEFORE   AFTER        BEFORE   AFTER    REPAIRED     REPAIR  NETWORK       FLOW     OF ARC
NUMBER   FROM     TO    STRIKE   STRIKE       STRIKE   STRIKE    ON DAY       COST     FLOW        COST     STRIKES
    1      A       K      900      831          900      831        2           35     4931        60658        1
    2      A       K      831      767          831      767        4           67     4867        59506        2
    3      A       K      767      708          767      708        5           96     4808        58444        3
    4      A       K      708      654          708      654        5          123     4754        57472        4
    5      A       K      654      604          654      604        7          148     4704        56572        5
    6      C       E      900      848          900      848        2          183     4652        56052        1
    7      A       K      604      558          604      558        8          206     4606        55224        6
    8      C       E      848      799          848      799        3          239     4557        54734        2
    9      A       K      558      515          558      515        9          260     4514        53960        7
   10      C       E      799      752          799      752        3          291     4467        53490        3
   11      C       E      752      708          752      708        4          320     4423        53050        4
   12      A       K      515      475          515      475       10          360     4383        52330        8
   13      C       E      708      667          708      667        5          367     4342        51920        5
   14      E       H      800      753          800      753        2          390     4295        51403        1
   15      A       K      475      438          475      438       10          408     4258        50737        9
   16      C       E      667      628          667      628        6          434     4219        50347        6
   17      E       H      753      709          753      709        2          456     4175        49863        2
   18      A       K      438      404          438      404       11          473     4141        49251       10
   19      C       E      628      591          628      591        6          497     4104        48881        7
   20      E       H      709      668          709      668        3          518     4063        48430        3
   21      C       E      591      557          591      557        7          541     4029        48090        8
   22      A       K      404      373          404      373       12          557     3998        47532       11
   23      D       G     1000      951         1000      951        1          572     3949        46846        1
   24      E       H      668      629          668      629        3          591     3910        46417        4
   25      C       E      557      525          557      525        7          613     3878        46097        9
   26      D       G      951      905          951      905        2          627     3832        45453        2
   27      E       H      629      592          629      592        4          645     3795        45046        5
   28      A       K      373      344          373      344       12          659     3766        44524       12
   29      C       E      525      496          525      496        9          679     3737        44214       10
   30      D       G      905      861          905      861        2          692     3691        43509        3
   31      E       H      592      558          592      558        6          709     3657        43224        5
   32      D       G      861      819          861      819        2          722     3615        42636        4
   33      C       E      496      465          496      465        8          741     3596        42346       11
   34      A       K      344      318          344      318       13          754     3560        41878       13
   35      E       H      558      526          558      526        4          770     3528        41526        7
   36      D       G      819      779          819      779        3          782     3488        40955        5
   37      C       E      465      438          465      438        9          800     3461        40596       12
   38      E       H      526      495          526      495        5          815     3430        40255        9
```

```
STRIKES TARGETED FOR DAY  1.
                          ARC CAPACITY          ARC FLOW       TO BE       CUM.   MAXIMUM     NETWORK   NUMBER
MISSION                 BEFORE  AFTER        BEFORE  AFTER   REPAIRED    REPAIR  NETWORK      FLOW     OF ARC
NUMBER   FROM    TO     STRIKE  STRIKE       STRIKE  STRIKE   ON DAY      COST     FLOW       COST     STRIKES
  39      A      K       318     294          318     294       13        827      3406      39923       14
  40      D      G       779     741          779     741        3        838      3368      39391        6
  41      C      E       438     412          438     412        9        855      3342      39131       13
  42      E      H       495     466          495     466        5        860      3313      38812        9
  43      D      G       741     705          741     705        3        880      3277      38308        7
  44      A      K       294     271          294     271       14        891      3254      37894       15
  45      C      E       412     388          412     388       10        907      3230      37654       14
  46      D      G       705     671          705     671        4        917      3194      37178        8
  47      E      H       466     439          466     439        6        931      3169      36981       10
  48      A      K       271     250          271     250       14        941      3148      36503       16
  49      C      E       388     365          388     365       10        956      3125      36273       15
  50      D      G       671     638          671     638        4        966      3092      35811        9

           RECAP FOR DAY  1:
           ****************

                                       TOTAL NUMBER      TOTAL NUMBER
            ARC                        ARC STRIKES       ARC STRIKES
            NAME      FROM      TO     FOR DAY  1        FOR CAMPAIGN
            C E        C        E          15                15
            D G        D        G           9                 9
            E H        E        H          10                10
            A K        A        K          16                16

            NUMBER STRIKES THIS DAY      :        50
            NUMBER STRIKES THIS CAMPAIGN :        50
```

```
ALL ARC REPAIR SCHEDULED FOR DAY  2 HAS BEEN COMPLETED.
RESULTING FLOW PATTERN IS AS FOLLOWS:

                                LOWER        UPPER                   MAX.       MAX.                   ARC
ARC      FROM       TO          FLOW         FLOW       FLOW       REPAIR     REPAIR      ARC         VULNER.
NAME     NODE       NODE        BOUND        BOUND      COST        COST       TIME       FLOW        PARAM.
ZZAA      ZZ         AA           0          100000     -1000          0          0       3425        0.0
AAA       AA         A            0          100000         0          0          0        701        0.0
AAB       AA         B            0          100000         0          0          0        924        0.0
AAC       AA         C            0          100000         0          0          0       1027        0.0
AAD       AA         D            0          100000         0          0          0        773        0.0
H  ZZ     H          ZZ           0          100000         0          0          0       1525        0.0
I  ZZ     I          ZZ           0          100000         0          0          0       1100        0.0
J  ZZ     J          ZZ           0          100000         0          0          0        400        0.0
L  ZZ     L          ZZ           0          100000         0          0          0        400        0.0
B  E      B          E            0            1200         5        200          5        924        0.070
C  E      C          E            0             427         3        600         15        427        0.060
C  G      C          G            0             600         2        500         13        600        0.030
D  G      D          G            0             773         4        300          8        773        0.050
E  H      E          H            0             524         6        400         10        524        0.060
E  F      E          F            0             500         9        800         20        500        0.020
F  G      G          F            0             900         3        500         13        273        0.010
F  H      F          H            0             800         5        750         20        800        0.030
F  I      F          I            0            1100         7        300          8        400        0.040
G  I      G          I            0             700         5        200          5        700        0.060
G  J      G          J            0             400         6        400         11        400        0.050
A  K      A          K            0             301         6        450         18        301        0.090
A  E      A          E            0             400         3        550         14        400        0.050
K  E      E          K            0             300         2        350         20        300        0.030
K  L      K          L            0             400         7        450         25        400        0.030
K  H      K          H            0            1000        12        550         18        201        0.010

FLOW COST  :    40174
THROUGHPUT :     3425
```

```
STRIKES TARGETED FOR DAY  2.
                        ARC CAPACITY          ARC FLOW       TO BE      CUM.   MAXIMUM    NETWORK   NUMBER
MISSION                BEFORE  AFTER      BEFORE   AFTER   REPAIRED   REPAIR  NETWORK     FLOW     OF ARC
NUMBER   FROM   TO     STRIKE  STRIKE     STRIKE   STRIKE   ON DAY     COST    FLOW       COST     STRIKES
    1     E     H       524     493        524      493       6         981    3394      39833       11
    2     D     G       773     735        773      735       4         992    3356      39301       10
    3     A     K       301     278        301      278      14        1004    3333      38887       17
    4     E     H       493     464        493      464       6        1018    3304      38558       12
    5     C     E       427     402        427      402      10        1035    3270      38318       16
    6     D     G       735     699        735      699       4        1046    3243      37814       11
    7     D     G       699     665        699      665       5        1056    3209      37338       12
    8     E     H       464     437        464      437       7        1070    3182      37041       13
    9     C     E       402     379        402      379      11        1086    3159      36811       17
   10     A     K       278     257        278      257      15        1097    3138      36433       18
   11     D     G       665     633        665      633       5        1107    3106      35985       13
   12     C     E       379     357        379      357      11        1122    3084      35765       18
   13     E     H       437     412        437      412       7        1135    3059      35490       14
   14     A     K       257     237        257      237      15        1145    3039      35130       19
   15     D     G       633     602        633      602       5        1154    3008      34696       16
   16     C     E       357     336        357      336      11        1168    2987      34496       10
   17     E     H       412     388        412      388       7        1180    2963      34222       15
   18     C     G       600     582        600      582       2        1195    2945      34006        1
   19     D     G       602     573        602      573       6        1204    2916      33600       15
   20     C     E       336     316        336      316      12        1217    2896      33400       20
   21     A     K       237     219        237      219      15        1226    2873      33076       20
   22     E     H       388     365        388      365       7        1237    2855      32923       16
   23     C     G       582     565        582      565       3        1251    2838      32619        2
   24     D     G       573     545        573      545       6        1259    2810      32227       16
   25     C     G       565     548        565      548       3        1273    2793      32051        3
   26     D     G       545     518        545      518       6        1281    2766      31781       17
   27     A     K       219     202        219      202      15        1289    2749      31475       21
   28     C     G       548     532        548      532       3        1302    2733      31347        4
   29     E     H       365     344        365      344       8        1313    2712      31116       17
   30     C     E       316     298        316      298      12        1325    2694      30940       21
   31     C     G       532     516        532      516       4        1338    2678      30812        5
   32     E     K       300     291        300      291       3        1348    2669      30641        1
   33     E     K       291     282        291      282       3        1358    2660      30470        2
   34     D     G       518     493        518      493       6        1366    2635      30220       18
   35     E     H       344     324        344      324       9        1376    2615      30000       18
   36     C     G       516     501        516      501       4        1390    2600      29890        6
   37     C     E       298     281        298      281      12        1401    2583      29744       22
   38     A     K       202     186        202      186      16        1409    2567      29456       22
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```
STRIKES TARGETED FOR DAY  2.
                        ARC CAPACITY        ARC FLOW      TO BE        CUM.   MAXIMUM    NETWORK   NUMBER
MISSION                BEFORE   AFTER     BEFORE   AFTER  REPAIRED   REPAIR  NETWORK     FLOW     OF ARC
NUMBER   FROM    TO   STRIKE   STRIKE    STRIKE   STRIKE   ON DAY     COST     FLOW       COST    STRIKES
  39      C      G      501      486       501      486       4       1421     2552      29336       7
  40      D      G      493      469       493      469       6       1428     2529      29026      19
  41      E      H      324      305       324      305       9       1437     2509      28987      19
  42      E      K      292      274       292      274       4       1447     2501      28735       3
  43      C      E      281      265       281      265      13       1458     2485      28607      23
  44      C      G      486      472       486      472       5       1470     2471      28495       9
  45      E      K      274      266       274      266       4       1479     2453      28343       4
  46      C      G      472      458       472      458       5       1491     2449      28231       9
  47      D      G      469      446       469      446       7       1498     2426      28001      20
  48      E      K      266      258       266      258       5       1507     2418      27849       5
  49      C      G      458      444       458      444       5       1518     2404      27737      10
  50      E      K      258      250       258      250       5       1527     2396      27585       6

          RECAP FOR DAY  2:
          *****************

                                          TOTAL NUMBER      TOTAL NUMBER
            ARC                           ARC STRIKES       ARC STRIKES
            NAME      FROM       TO       FOR DAY  2        FOR CAMPAIGN
            C  E       C         E              8                 23
            C  G       C         G             10                 10
            D  G       D         G             11                 20
            E  H       E         H              9                 19
            A  K       A         K              6                 22
            K  E       E         K              6                  6

            NUMBER STRIKES THIS DAY      :          50
            NUMBER STRIKES THIS CAMPAIGN :         100
```

```
ALL ARC REPAIR SCHEDULED FOR DAY  3 HAS BEEN COMPLETED.
RESULTING FLOW PATTERN IS AS FOLLOWS:

                               LOWER       UPPER                 MAX.        MAX.                   ARC
ARC      FROM        TO        FLOW        FLOW        FLOW     REPAIR      REPAIR       ARC       VULNER.
NAME     NODE       NODE       BOUND       BOUND       COST      COST        TIME        FLOW      PARAM.
ZZAA      ZZ         AA          0        100000      -1000        0           0         2783      0.0
AAA       AA         A           0        100000          0        0           0          636      0.0
AAB       AA         B           0        100000          0        0           0          754      0.0
AAC       AA         C           0        100000          0        0           0          820      0.0
AAD       AA         D           0        100000          0        0           0          573      0.0
H  ZZ     H          ZZ          0        100000          0        0           0         1290      0.0
I  ZZ     I          ZZ          0        100000          0        0           0          726      0.0
J  ZZ     J          ZZ          0        100000          0        0           0          267      0.0
L  ZZ     L          ZZ          0        100000          0        0           0          400      0.0
B  E      B          E           0          1200          5      200           5          754      0.070
C  F      C          F           0           326          3      600          15          326      0.060
C  G      C          G           0           494          2      500          13          494      0.030
D  G      D          G           0           573          4      300           8          573      0.050
E  H      E          H           0           387          6      400          10          387      0.060
E  F      E          F           0           500          9      800          20          500      0.020
F  H      F          H           0           800          5      750          20          800      0.030
F  I      F          I           0          1100          7      300           8           26      0.040
G  I      G          I           0           700          5      200           5          700      0.050
G  J      G          J           0           400          6      400          11          267      0.050
A  K      A          K           0           236          6      450          19          236      0.080
A  E      A          E           0           400          3      550          14          400      0.050
K  F      E          K           0           267          2      350          20          267      0.030
K  L      K          L           0           400          7      450          25          400      0.030
K  H      K          H           0          1000         12      550          18          103      0.010


FLOW COST  :     31920
THROUGHPUT :      2783
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```
STRIKES TARGETED FOR DAY  3.
                         ARC  CAPACITY           ARC  FLOW       TO BE        CUM.    MAXIMUM     NETWORK    NUMBER
MISSION                 BEFORE   AFTER        BEFORE    AFTER   REPAIRED     REPAIR  NETWORK      FLOW      OF ARC
NUMBER    FROM    TO    STRIKE   STRIKE       STRIKE    STRIKE   ON DAY       COST     FLOW        COST     STRIKES
   1       A      K      236      218          236       218       17         1536     2765       31596       23
   2       F      H      387      364          387       364        9         1547     2742       31343       20
   3       D      G      573      545          573       545        7         1555     2714       31053       21
   4       C      E      326      307          326       307       13         1568     2695       30873       24
   5       D      G      545      518          545       518        7         1576     2668       30603       22
   6       A      K      218      201          218       201       17         1584     2651       30297       24
   7       F      H      364      343          364       343        9         1595     2630       30066       21
   8       C      E      307      289          307       289       13         1607     2612       29908       25
   9       D      G      518      493          518       493        7         1615     2587       29658       23
  10       E      H      343      323          343       323        9         1625     2567       29438       22
  11       C      E      289      272          289       272       14         1636     2550       29302       26
  12       C      G      494      470          494       470        6         1648     2535       29192       11
  13       D      G      493      469          493       469        7         1655     2511       28942       24
  14       A      K      201      186          201       186       17         1663     2495       28672       25
  15       E      H      323      304          323       304        9         1672     2477       28463       23
  16       C      G      470      465          470       465        6         1684     2463       28351       12
  17       C      E      272      256          272       256       14         1695     2447       28223       27
  18       D      G      460      446          459       446                  1702     2424       27903       25
  19       C      G      465      451          465       451        6         1713     2410       27891       13
  20       E      K      267      259          267       259        5         1722     2402       27720        7
  21       E      K      259      251          259       251        6         1731     2394       27577        8
  22       E      H      304      286          304       286        9         1740     2376       27370       24
  23       A      K      186      172          186       172       18         1747     2362       27127       26
  24       F      E      500      490          500       490        3         1763     2352       26937        1
  25       F      E      490      480          490       480        4         1779     2342       26747        2
  26       D      G      446      424          446       424        8         1786     2320       26527       26
  27       F      E      480      470          480       470        4         1801     2310       26337        3
  28       C      G      451      438          451       438        7         1812     2297       26233       14
  29       C      E      256      241          256       241       14         1822     2282       26113       28
  30       C      G      438      425          438       425        7         1833     2269       26009       15
  31       C      G      425      412          425       412        7         1843     2256       25905       16
  32       E      H      286      269          286       269       10         1851     2239       25718       25
  33       D      G      424      403          424       403        8         1857     2218       25508       27
  34       F      K      251      244          251       244        7         1866     2211       25375        9
  35       A      K      172      159          172       159       18         1873     2198       25141       27
  36       E      K      244      237          244       237        7         1881     2191       25028       10
  37       F      E      470      461          470       461        5         1896     2182       24957        4
  38       C      E      241      227          241       227       14         1905     2168       24745       29
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STRIKES TARGETED FOR DAY  3.
                              ARC CAPACITY           ARC FLOW         TO BE       CUM.   MAXIMUM     NETWORK   NUMBER
MISSION                      BEFORE   AFTER      BEFORE   AFTER    REPAIRED   REPAIR  NETWORK      FLOW     OF ARC
NUMBER    FROM    TO         STRIKE   STRIKE     STRIKE   STRIKE    ON DAY     COST    FLOW        COST     STRIKES
  39       E      F           461      452        461      452        5        1920    2159       24574       5
  40       E      K           237      230        237      230        8        1928    2152       24476      11
  41       E      F           452      443        452      443        5        1942    2143       24305       6
  42       C      G           412      400        412      400        7        1952    2131       24209      17
  43       E      K           230      223        230      223        8        1960    2124       24111      12
  44       D      G           403      393        403      393        8        1966    2104       23911      28
  45       E      F           443      434        443      434        6        1980    2095       23740       7
  46       E      H           269      253        269      253       10        1988    2079       23564      26
  47       C      G           400      388        400      388        8        1998    2067       23468      18
  48       E      F           434      425        434      425        6        2012    2058       23297       9
  49       E      K           223      216        223      216        9        2020    2051       23199      13
  50       A      K           159      147        159      147       18        2025    2039       23043      29

      RECAP FOR DAY  3:
      *****************

                                        TOTAL NUMBER     TOTAL NUMBER
        ARC                             ARC STRIKES      ARC STRIKES
        NAME      FROM       TO         FOR DAY  3       FOR CAMPAIGN
        C F        C         F                5               22
        C G        C         G                8               18
        D G        D         G                9               29
        E H        E         H                7               26
        E F        E         F                8                9
        A K        A         K                6               28
        K E        E         K                7               13

        NUMBER STRIKES THIS DAY      :        50
        NUMBER STRIKES THIS CAMPAIGN :       150
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ALL ARC REPAIR SCHEDULED FOR DAY  4 HAS BEEN COMPLETED.
RESULTING FLOW PATTERN IS AS FOLLOWS:

                                LOWER      UPPER             MAX.     MAX.               ARC
ARC      FROM      TO           FLOW       FLOW     FLOW    REPAIR   REPAIR    ARC       VULNER.
NAME     NODE      NODE         BOUND      BOUND    COST     COST     TIME     FLOW      PARAM.
ZZAA      ZZ        AA            0       100000   -1000       0        0      2446      0.0
AAA       AA        A             0       100000       0       0        0       597      0.0
AAB       AA        B             0       100000       0       0        0       618      0.0
AAC       AA        C             0       100000       0       0        0       722      0.0
AAD       AA        D             0       100000       0       0        0       509      0.0
H  ZZ     H         ZZ            0       100000       0       0        0      1102      0.0
I  ZZ     I         ZZ            0       100000       0       0        0       700      0.0
J  ZZ     J         ZZ            0       100000       0       0        0       244      0.0
L  ZZ     L         ZZ            0       100000       0       0        0       400      0.0
B  E      B         E             0         1200       5     200        5       618      0.070
C  F      C         F             0          287       3     600       15       287      0.060
C  G      C         G             0          435       2     500       13       435      0.030
D  G      D         G             0          509       4     300        8       509      0.050
E  H      E         H             0          334       6     400       10       334      0.060
E  F      E         F             0          453       0     800       20       453      0.020
F  H      F         H             0          800       5     750       20       740      0.030
G  I      G         I             0          700       5     200        5       700      0.060
G  J      G         J             0          400       6     400       11       244      0.050
A  K      A         K             0          197       6     450       18       197      0.080
A  F      A         F             0          400       3     550       14       400      0.050
K  E      E         K             0          231       2     350       20       231      0.030
K  L      K         L             0          400       7     450       25       400      0.030
K  H      K         H             0         1000      12     550       18        28      0.010

FLOW COST   :    27582
THROUGHPUT  :     2446
```

```
STRIKES TARGETED FOR DAY  4.
                            ARC CAPACITY          ARC FLOW      TO BE       CUM.    MAXIMUM      NETWORK    NUMBER
MISSION                   BEFORE   AFTER       BEFORE   AFTER   REPAIRED   REPAIR   NETWORK        FLOW     OF ARC
NUMBER    FROM    TO      STRIKE   STRIKE      STRIKE   STRIKE   ON DAY     COST     FLOW         COST     STRIKES
   1       D      G        509      484         509      484       8        2033     2421        27332       29
   2       C      F        287      270         287      270      15        2044     2404        27196       30
   3       E      H        334      315         334      315      10        2054     2385        26987       27
   4       D      G        484      460         484      460       8        2061     2361        26747       30
   5       A      K        197      182         197      182      13        2069     2346        26477       29
   6       C      F        270      254         270      254      15        2079     2330        26249       31
   7       E      H        315      297         315      297      10        2088     2312        26151       28
   8       D      G        460      438         460      438       9        2095     2290        25931       31
   9       A      K        182      168         182      168      19        2102     2276        25634       30
  10       C      F        254      239         254      239      15        2112     2261        25544       32
  11       E      H        297      280         297      280      11        2121     2244        25377       29
  12       C      G        435      422         435      422       8        2132     2231        25273       19
  13       D      G        438      417         438      417       9        2138     2210        25053       32
  14       A      K        168      155         168      155      10        2144     2197        24894       31
  15       E      H        280      264         280      264      11        2152     2181        24718       30
  16       C      F        239      225         239      225      15        2161     2167        24605       33
  17       D      G        417      397         417      397       9        2167     2147        24406       33
  18       C      G        422      410         422      410       8        2177     2135        24310       20
  19       E      F        453      444         453      444       6        2191     2126        24139        9
  20       F      K        231      224         231      224       9        2199     2119        24041       14
  21       C      G        410      398         410      398       8        2209     2107        23945       21
  22       E      F        444      435         444      435       7        2223     2098        23776       10
  23       E      F        435      426         435      426       7        2237     2089        23603       11
  24       F      K        224      217         224      217      10        2245     2082        23505       15
  25       C      G        398      386         398      386       9        2255     2070        23409       22
  26       D      G        397      378         397      378       9        2261     2051        23219       34
  27       E      H        264      249         264      249      11        2269     2035        23054       31
  28       A      K        155      143         155      143      19        2275     2024        22898       32
  29       C      F        225      212         225      212      15        2284     2011        22794       34
  30       C      G        386      375         386      375       9        2294     2000        22706       23

              RECAP FOR DAY  4:
              *****************

                                            TOTAL NUMBER      TOTAL NUMBER
                 ARC                        ARC STRIKES       ARC STRIKES
                 NAME      FROM      TO     FOR DAY  4        FOR CAMPAIGN
                 C F        C        F             5               34
                 C G        C        G             5               23
                 D G        D        G             6               34
                 E H        E        H             5               31
                 E F        E        F             3               11
                 A K        A        K             4               32
                 K F        F        K             2               15

                 NUMBER STRIKES THIS DAY      :      30
                 NUMBER STRIKES THIS CAMPAIGN :     180
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ALL ARC REPAIR SCHEDULED FOR DAY  5 HAS BEEN COMPLETED.
RESULTING FLOW PATTERN IS AS FOLLOWS:

                              LOWER       UPPER                 MAX.       MAX.                   ARC
ARC      FROM      TO         FLOW        FLOW        FLOW      REPAIR     REPAIR      ARC        VULNER.
NAME     NODE      NODE       BOUND       BOUND       COST      COST       TIME        FLOW       PARAM.
ZZAA      ZZ        AA          0        100000     -1000          0          0        2406       0.0
AAA       AA        A           0        100000         0          0          0         593       0.0
AAB       AA        B           0        100000         0          0          0         616       0.0
AAC       AA        C           0        100000         0          0          0         694       0.0
AAD       AA        D           0        100000         0          0          0         503       0.0
H  ZZ     H         ZZ          0        100000         0          0          0        1081       0.0
I  ZZ     I         ZZ          0        100000         0          0          0         700       0.0
J  ZZ     J         ZZ          0        100000         0          0          0         225       0.0
L  ZZ     L         ZZ          0        100000         0          0          0         400       0.0
B  F      B         F           0          1200         5        200          5         616       0.070
C  F      C         F           0           272         3        600         15         272       0.060
C  G      C         G           0           422         2        500         13         422       0.030
D  G      D         G           0           503         4        300          8         503       0.050
E  H      E         H           0           330         6        400         10         330       0.060
E  F      E         F           0           453         9        800         20         453       0.020
F  H      F         H           0           800         5        750         20         725       0.030
G  I      G         I           0           700         5        200          5         700       0.060
G  J      G         J           0           400         6        400         11         225       0.050
A  K      A         K           0           193         6        450         18         193       0.080
A  F      A         F           0           400         3        550         14         400       0.050
K  F      F         K           0           233         2        350         20         233       0.030
K  L      K         L           0           400         7        450         25         400       0.030
K  H      K         H           0          1000        12        550         18          26       0.010


FLOW COST   :    27220
THROUGHPUT  :     2406
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```
STRIKES TARGETED FOR DAY  5.
                         ARC CAPACITY          ARC FLOW       TO BE       CUM.   MAXIMUM     NETWORK   NUMBER
MISSION                 BEFORE  AFTER       BEFORE  AFTER   REPAIRED    REPAIR  NETWORK      FLOW     OF ARC
NUMBER   FROM     TO    STRIKE  STRIKE      STRIKE  STRIKE   ON DAY      COST    FLOW        COST     STRIKES
   1      D       G      503     478         503     478        9        2301    2381       26970       35
   2      E       H      330     311         330     311       11        2311    2362       26761       32
   3      A       K      193     178         193     178       19        2319    2347       26491       33
   4      D       G      478     455         478     455        9        2325    2324       26261       36
   5      C       E      272     256         272     256       16        2336    2308       26123       35
   6      E       H      311     293         311     293       11        2345    2290       25935       33
   7      D       G      455     433         455     433       10        2352    2268       25715       37
   8      A       K      178     164         178     164       20        2359    2254       25478       34
   9      C       E      256     241         256     241       16        2369    2239       25358       36
  10      E       H      293     276         293     276       12        2379    2222       25171       34
  11      D       G      433     412         433     412       10        2386    2201       24961       38
  12      C       E      241     227         241     227       16        2393    2187       24840       37
  13      A       K      164     151         164     151       20        2399    2174       24680       35
  14      C       G      422     410         422     410        9        2409    2162       24584       24
  15      D       G      412     392         412     392       10        2415    2142       24384       39
  16      E       H      276     260         276     260       12        2423    2126       24208       35
  17      E       F      453     444         453     444        7        2437    2117       24037       12
  18      F       K      233     226         233     226       10        2445    2110       23939       16
  19      C       G      410     398         410     398        9        2455    2098       23843       25
  20      E       F      444     435         444     435        8        2469    2089       23672       13
  21      F       K      226     219         226     219       10        2477    2082       23574       17
  22      E       F      435     426         435     426        8        2491    2073       23403       14
  23      C       G      398     386         398     386       10        2501    2061       23307       26
  24      D       G      392     373         392     373       10        2507    2042       23117       40
  25      C       E      227     214         227     214       16        2516    2029       23013       38
  26      E       H      260     245         260     245       12        2524    2014       22848       36
  27      A       K      151     139         151     139       20        2530    2002       22692       36
  28      C       G      386     375         386     375       10        2540    1991       22604       27

          RECAP FOR DAY  5:
          *****************

                                          TOTAL NUMBER     TOTAL NUMBER
            ARC                           ARC STRIKES      ARC STRIKES
            NAME      FROM       TO       FOR DAY  5       FOR CAMPAIGN
            C  E       C         E             4                38
            C  G       C         G             4                27
            D  G       D         G             6                40
            E  H       E         H             5                36
            E  F       E         F             3                14
            A  K       A         K             4                36
            K  F       F         K             2                17

            NUMBER  STRIKES  THIS  DAY       :        28
            NUMBER  STRIKES  THIS  CAMPAIGN  :       209
```

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A Dynamic Model for the Allocation of Airstrikes Against a Lines-of-Communication Network.

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13. ABSTRACT

A computer model is presented for determining the daily allocation of airstrikes for interdicting a lines-of-communication (LOC) network assuming an exponential damage function. The allocation of sorties to arcs in the network is based on the assumptions that (1) flow of supplies is restricted by network capacity and (2) parameter values for each arc are independent. Upper and lower flow bounds, cost per unit flow, maximum repair time and cost, and the vulnerability parameter for each arc are required as input data. The model selects that arc to strike which maximizes the repair cost plus the product of the increase in minimum cost circulation flow and repair time. The procedure is programmed in daily cycles, allowing repair on interdicted arcs. A sample problem and all documentation necessary for duplicating the computer program are given.

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